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**October 21<sup>st</sup> Mathematics Focus Session**  
*Reflective Questions for Consideration Prior to the Session*

A.) In the **Mathematical Education of Teachers** <[http://cbmsweb.org/MET\\_Document/index.htm](http://cbmsweb.org/MET_Document/index.htm)> the Conference Board of the Mathematical Sciences provided the research and rationale supporting a re-visioning of our preparation of teachers of K-12 mathematics. For example, the MET document recommends the following amounts of mathematics coursework for prospective teachers:

- Prospective elementary grade teachers should be required to take at least 9 semester-hours (beyond general college coursework) on fundamental ideas of school mathematics for elementary teachers.
- Prospective middle grades teachers of mathematics should be required to take at least 21 semester-hours of mathematics, that includes at least 12 semester-hours on fundamental ideas of school mathematics appropriate for middle grades teachers.
- Prospective high school teachers of mathematics should be required to complete the equivalent of an undergraduate major in mathematics that includes a 6-hour capstone course connecting their college mathematics courses with high school mathematics.

Consider the level your program represents -- elementary, middle grades, secondary. How many hours of mathematics coursework are currently in your program of study addressing fundamental ideas of school mathematics for elementary school, middle grades, or high school teachers? If you had the flexibility of credit hours, how would you design the program and courses to meet or exceed this recommendation? Beyond mathematical content, what other specialized courses would you view as critical for inclusion in your programs of study?

B.) If you were asked to develop a course addressing formative assessment for mathematics, how would you design it at the elementary, middle grades, and high school levels?

C.) Attached is a DRAFT copy of the Pre-Service Mathematics Teaching Standards currently being considered by the State Board of Education. How do you plan to address these standards in your campus programs re-visioning efforts?

## Standards for Mathematics Teachers

What mathematical knowledge do teachers need to know? Content knowledge that pre-service teachers learn in mathematics education must strongly emphasize “mathematical knowledge for teaching.” This includes an understanding of the ways learners think, knowledge of didactic representations, the ability to make pedagogical judgments about students’ questions and solutions to mathematical problems, and the ability to make judgments about the mathematical quality of instructional materials. The advanced mathematics in a traditional college mathematics program, while important, may be “remote from the core content of the K-12 curriculum.”<sup>1</sup> It is important that the mathematics curriculum of a teacher licensure program include content necessary for teacher licensure candidates to develop deep understanding of the mathematics that *they will teach*.

An accredited teacher licensure program provides bridges from traditional mathematics content to the mathematics knowledge for teaching. Ideally, such bridges are integrated in mathematics coursework. However, given that this is not always possible, then other special courses may need to be designed specifically for pre-service teachers. Elementary grades teachers must be provided with *substantial* opportunities to focus on real numbers, data analysis and probability, geometry and measurement, and algebra. Similar and possibly more extensive opportunities are recommended for middle grade teachers, focusing on real and imaginary numbers with an emphasis on rational numbers and proportional reasoning, geometry, algebra, and data analysis and probability. Secondary mathematics teachers need *substantial* opportunities to address the mathematical knowledge for teaching 9-12 school mathematics. The focus needs to address knowledge of the mathematical understandings and skills that students acquire in elementary and middle school and how they affect learning in high school. In addition secondary mathematics teachers need a deep understanding of the fundamental mathematical ideas in grades 9-12 mathematics curricula and strong technical skills for application of those ideas (e.g., Algebra I/II, Geometry).

<sup>1</sup>Online introduction to Ball, D.L., Hill, H.C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, pp.14-22, 43-46. Retrieved April 19, 2008 from [http://www.aft.org/pubs-reports/american\\_educator/issues/fall2005/bond.htm](http://www.aft.org/pubs-reports/american_educator/issues/fall2005/bond.htm)

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author

Conference Board of the Mathematical Sciences. (2001). *The mathematical education of teachers*. Providence RI and Washington DC: American Mathematical Society and Mathematical Association of America. Retrieved April 19, 2008 from [http://www.cbmsweb.org/MET\\_Document/index.html](http://www.cbmsweb.org/MET_Document/index.html)

National Mathematics Advisory Panel. (2008). *Foundations for success: Final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education

**Standard 2: 21<sup>st</sup> Century Teacher candidates have the knowledge and understanding of mathematical conventions and processes skills relative to: Number sense, numeration, numerical operations, and algebraic thinking; spatial sense, measurement and geometry; patterns, relationships, and functions; and data analysis, probability and statistics. *Mathematics***

**A. Problem solving, reasoning and proof, communication, connection, and representation.**

- help students develop problem solving skills that involve building new mathematical knowledge, applying and adapting a variety of appropriate strategies in order to monitor and reflect on the process of mathematical problem solving.
- facilitate instruction using reasoning and proof which involves making and investigating mathematical conjecture, developing and evaluating mathematical arguments and proofs, and selecting and using various types of reasoning and methods of proof.
- help students communicate mathematical thinking coherently and clearly, analyzing and evaluating mathematical thinking and strategies of others, and using the language of mathematics to express mathematical ideas precisely
- help students make connections regarding how mathematics ideas interconnect and build on one another to produce a coherent whole and recognizing and applying mathematics in context outside of mathematics.
- help students develop representations which involves selecting, applying, and translating among mathematical representations to solve problems and using representations to organize, record, and communicate mathematical ideas.

**B. Number sense, numeration, and numerical operations**

- understand numbers, ways of representing numbers, relationships among numbers and number systems
- understand meanings of operations and how they relate to one another
- compute fluently and make reasonable estimates
- possess a large repertoire of place value knowledge and the its use throughout computation, alternative computational algorithms, and knowledge of fractions.

**C. Spatial sense, measurement and geometry**

- understand and analyze the characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- understand how to specify locations and describe special relationships using coordinate geometry and other representational systems.
- understand and apply transformations and use symmetry to analyze mathematical situations
- understand how to use visualization, spatial reasoning, and geometric modeling to solve problems.
- understand measurable attributes of objects and the units, systems, and processes of measurement
- understand how to apply appropriate techniques, tools, and formulas to determine measurement.

**D. Patterns, relationships, and functions and algebraic thinking**

- understand patterns, relations and functions
- understand how to represent and analyze mathematical situations and structures using algebraic symbols
- understand how to use mathematical models to represent and understand quantitative relationships
- are able to analyze change in various contexts

The Mathematics Standard for K-5 Teaching  
To be hyperlinked

**E. Data analysis, probability and statistics**

- understand how to formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
- understand how to select and use appropriate statistical methods to analyze data
- know how to develop and evaluate inferences and predictions that are based on data
- understand and apply basic concepts of probability

**F. Mathematical instructional strategies and tools**

- understand ways to use technology to explore mathematical concepts.
- use appropriate math manipulatives and representations

**Standard 1: Number sense, numeration, numerical operation, and algebraic thinking**

Teacher candidates possess the mathematical knowledge needed to enable students to understand numbers, ways of representing numbers, and relationships among numbers and number systems and to enable students to understand meanings of operations and how they relate to one another. Candidates enable students to develop computational fluency and to make reasonable estimates. At the middle and secondary grade levels, teacher candidates need the mathematical knowledge to enable students to transfer their understanding of numbers and number operations to symbolic expressions involving variables.

**Middle School**

- Demonstrate conceptual understanding of rational numbers, facility in operating with rational numbers, and the ability to move flexibly among different representations of rational numbers (i.e. fractions, decimals, and percents).
- Understand and explain the distinctions among whole numbers, integers, rational numbers, and real numbers, and use properties (e.g. commutative, associative, distributive, and inverse) of these number systems.
- Understand and use mental computation and computational estimation.
- Understand and explain fundamental ideas of number theory (e.g. factors, multiples, divisibility, and primes) as they apply to middle school mathematics.
- Understand and extend the concepts of place value to make sense of large and small numbers, and use scientific notation.
- Demonstrate conceptual understanding of proportionality and facility in solving proportional reasoning problems, with an emphasis on multiplicative reasoning.

**High School**

- Understand and apply the mathematics of natural, integer, rational, real, and complex number systems.
- Understand and apply the mathematics of algebraic structures (e.g. groups, rings and fields) and rules for operations on expressions, equations, inequalities, vectors and matrices.
- Demonstrate skill in using algebra to model real-world applications.

**Standard 2: Spatial sense, measurement and geometry**

Teacher candidates possess the mathematical knowledge needed to enable students to analyze the characteristics and properties of 2- and 3-dimensional geometric shapes; to develop mathematical arguments about geometric relationships; to understand units, processes of measure, and measurable attributes of objects; and to apply appropriate techniques, tools, and formulas to determine measurements. They enable students to develop the visualization, spatial reasoning, and geometric modeling to solve problems. Teacher candidates particularly at middle and secondary grade levels need the mathematical knowledge to enable students to use coordinate geometry in solving problems, to understand concepts of symmetry, and to apply transformations.

**Middle School**

- Identify basic characteristics and properties of common 2- and 3-dimensional shapes.
- Visualize and solve problems involving 2- and 3-dimensional shapes.
- Make conjectures about geometric shapes and prove or disprove the conjectures.
- Understand and use rigid motions (i.e. reflections, rotations, and translations) in the plane.
- Understand how similar figures result from dilation and the role of proportional relationships in determining similarity.
- Connect geometry to other mathematical topics (e.g. algebra and Pythagorean Theorem) and to nature and art.

- Understand and derive appropriate techniques, tools, and formulas to determine measurements.
- Choose appropriate tools, units, and systems for measuring.

### High School

- Understand core concepts and principles of Euclidean geometry in the plane and space.
- Use axiomatic reasoning and demonstrate facility with proof.
- Understand and apply the use of coordinates in 2- and 3-dimensional geometry, vectors and transformations, including matrix representations of transformations.
- Understand and apply trigonometry from a geometric perspective and demonstrate skill in using trigonometry to solve problems.

### Standard 3: **Patterns, relationships, and functions**

**Teacher candidates possess the mathematical knowledge needed to enable students to understand patterns, relations, and functions. This includes the use of algebraic symbols to represent and analyze mathematical situations, the use of mathematical models to represent and understand quantitative relationships, and the analysis of “change” in various contexts.**

### Middle School

- Understand the critical importance of the *concept* of variable and the use of variables in expressing functional relationships
- Understand and use algebra as a symbolic language; as a problem-solving tool; as generalized arithmetic; as a study of functions, relations, and variation; and as a way of modeling physical situations.
- Understand functions, including the abilities to read, interpret and create graphs, formulas (in closed and recursive forms), and tables for particular classes of functions.
- Understand linearity and how linear functions can illustrate proportional relationships.
- Recognize patterns of change associated with linear, quadratic, and exponential functions.
- Demonstrate algebraic skills and be able to provide rationales for common algebraic procedures.

### High School

- Understand and move flexibly among algebraic representations (e.g. tables, graphs, or formulas).
- Understand and recognize patterns in data that are modeled by important classes of functions.
- Understand and perform transformations of functions by arithmetically combining, composing, and inverting.
- Demonstrate and apply knowledge of important classes of functions (e.g., polynomial, exponential and logarithmic, rational, and periodic), including the effect of changing parameters within these classes of functions.
- Use functions to solve problems in calculus, linear algebra, geometry, statistics, and discrete mathematics.

#### **Standard 4: Data analysis, probability and statistics**

**Teacher candidates possess the mathematical knowledge needed to enable students to formulate questions that can be addressed with data, along with the necessary skills to collect, organize, and display relevant data to answer those questions. They enable students to select and use appropriate statistical methods to analyze data, to understand and apply basic concepts of probability, and to develop and evaluate inferences and predictions that are based on data.**

##### **Middle School**

- Engage in data investigations, including formulating questions and collecting data to answer questions.
- Understand and use a variety of ways to analyze variability in data distributions, including the use of summary statistics (e.g. measures of spread and center) and data representations (e.g. histograms, box plots, and scatter plots).
- Understand and apply basic concepts of experimental and theoretical probability.
- Understand principles of counting, including combinations, permutations and the Fundamental Counting Principle.
- Draw conclusions, generalizations and/or predictions that involve measures of uncertainty by applying basic concepts of probability.

##### **High School**

- Engage in data investigations, including formulating questions and collecting data to answer questions.
- Understand and use standard techniques for organizing, displaying and analyzing univariate data, with the ability to detect patterns and departures from patterns.
- Understand and use standard techniques for displaying and analyzing bivariate data (e.g. scatter plots, correlation and regression).
- Understand and use theory and simulations to study probability distributions
- Use probability models to draw conclusions from data and measure the uncertainty of those conclusions (e.g. confidence intervals and hypothesis tests).
- Understand and use basic rules and knowledge of probability such as conditional probability and independence, and develop skill in calculating probabilities associated with these concepts.
- Understand and use basic concepts of discrete mathematics (e.g. graph theory, combinatorics, iteration and recursion, modeling).

#### **Standard 5: Mathematical process skills**

**Teacher candidates possess the mathematical knowledge needed to enable students to develop skills in problem solving, making connections between various branches of mathematics, reasoning and proof, and communication and representation of mathematical ideas.**

##### **Middle School**

- Use problem solving to build new mathematical knowledge, apply and adapt a variety of appropriate strategies to solve problems, and monitor and reflect on the process of mathematical problem solving.
- Use reasoning and proof to make and investigate mathematical conjectures, develop and evaluate mathematical arguments and proofs, and select and use various types of reasoning and methods of proof
- Communicate mathematical thinking coherently and clearly, analyze and evaluate mathematical thinking and strategies of others, and use the language of mathematics to express mathematical ideas precisely.
- Make connections by understanding how mathematics ideas interconnect and by applying mathematics in context outside of mathematics.
- Use representations to organize, record, and communicate mathematical ideas.

### High School

- Use algebraic reasoning effectively for problem solving and proof in number theory, geometry, discrete mathematics, and statistics.
- Judge the reasonableness of numerical computations and their results.
- Judge the meaning, utility, and reasonableness of the results of symbolic manipulations, including those carried out by technology.

### Standard 6: **Mathematical tools**

**Teacher candidates must be versed in the appropriate use of mathematical tools and manipulatives.**

### Middle School

- Understand appropriate use of technology (e.g. graphing calculators, computer algebra systems, dynamic drawing tools, spreadsheets, or statistical graphing software) to explore algebraic, geometric and data analysis concepts.
- Use appropriate math manipulatives (e.g., algebra tiles, computer virtual manipulatives, or computer applets) to clarify and develop mathematical concepts.

### High School

- Understand appropriate use of technology (e.g. graphing calculators, computer algebra systems, dynamic drawing tools, spreadsheets, or statistical graphing software) to explore algebraic, geometric and data analysis concepts.
- Use appropriate math manipulatives (e.g., algebra tiles, computer virtual manipulatives, or computer applets) to clarify and develop mathematical concepts.