COMP 785 Advanced Algorithms Fall 1999 Quiz 2—<u>Solutions</u>

1. Rank the following functions by order of growth; that is, find an arrangement g_1, g_2, …, g_6 of the functions satisfying $g_1 = \Omega(g_2)$, $g_2 = \Omega(g_3)$, …, $g_5 = \Omega(g_6)$. Partition your list into equivalence classes such that $f(n)$ and $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n))$. Show your work or explain when appropriate.

$$n^{3/2} \quad n \lg n \quad 16^{n/4} \quad n + 3n^{3/2} \quad 3^n \quad n \log_{10}^2 n$$

Answer

Note that

$16^{n/4} = (16^{1/4})^n = ((16^{1/2})^{1/2})^n = (4^{1/2})^n = 2^n$

$n + 3n^{3/2} = 3n^{3/2} + n = \Theta(n^{3/2})$

So the ranking is

$$n \lg n \quad n \log_{10}^2 n \quad \begin{matrix} n^{3/2} \\ n + 3n^{3/2} \end{matrix} \quad 16^{n/4} \quad 3^n$$

2. Show that the following relations are true. (You may use limits or use the definitions in terms of constants.)

a. $9^{\log_3 n} = \Theta(3n^2 + 2n + 5)$

Answer

First note that

$$9^{\log_3 n} = 3^{2\log_3 n} = (3^{\log_3 n})^2 = n^2$$

Then:

$$\lim_{n\to\infty} \frac{n^2}{3n^2 + 2n + 5} = \lim_{n\to\infty} \frac{n^2}{3n^2} = \frac{1}{3}, \text{ a non-zero constant}$$

b. $2^{2n} = \Omega(3^n)$

Answer

First note that

$2^{2n} = (2^2)^n = 4^n$

Then

$$\lim_{n\to\infty} \frac{3^n}{4^n} = \lim_{n\to\infty} \left(\frac{3}{4}\right)^n = 0, \text{ a (possibly 0) constant}$$