

COMP 681 Formal Methods Spring 2009 Recitation 3—Solutions

1. Prove the following using a transformational proof. Justify each step. Show all substitutions used to derive appropriate instances of the rules.

$$p \vee q \Rightarrow r \langle \equiv \rangle (p \Rightarrow r) \wedge (q \Rightarrow r)$$

Answer

$$p \vee q \Rightarrow r$$

$$\langle \equiv \rangle \neg(p \vee q) \vee r \quad \text{Law of Implication } \{p / p \vee q, q / r\}$$

$$\langle \equiv \rangle \neg p \wedge \neg q \vee r \quad \text{De Morgan's Law}$$

$$\langle \equiv \rangle r \vee \neg p \wedge \neg q \quad \text{Commutativity of } \vee \{p / \neg p \vee \neg q, q / r\}$$

$$\langle \equiv \rangle (r \vee \neg p) \wedge (r \vee \neg q) \quad \text{Distributive Law } (\vee \text{ over } \wedge) \{p / r, q / \neg p, r / \neg p\}$$

$$\langle \equiv \rangle (\neg p \vee r) \wedge (\neg q \vee r) \quad \text{Commutativity of } \vee (2\times) \{p / r, q / \neg p\} \text{ and } \{p / r, q / \neg q\}$$

$$\langle \equiv \rangle (p \Rightarrow r) \wedge (q \Rightarrow r) \quad \text{Law of Implication } (2\times) \{q / r\} \text{ and } \{p / q, q / r\}$$

2. Prove the following using a transformational proof. Justify each step, but you need not show the substitution used to derive the appropriate instance of the laws. You may assume implicit associativity and commutativity of \wedge and \vee , and you may use generalized forms of the laws.

$$(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge p \langle \equiv \rangle p \wedge q \wedge r$$

Answer

$$(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge p$$

$$\langle \equiv \rangle \neg(p \vee q) \wedge (q \Rightarrow r) \wedge p \quad \text{Law of Implication}$$

$$\langle \equiv \rangle \neg(p \vee q) \wedge (\neg q \vee r) \wedge p \quad \text{Law of Implication}$$

$$\langle \equiv \rangle p \wedge \neg(p \vee q) \wedge (\neg q \vee r) \quad \text{Commutativity of } \wedge$$

$$\langle \equiv \rangle (p \wedge \neg p \vee p \wedge q) \wedge (\neg q \vee r) \quad \text{Distributive Law } (\wedge \text{ over } \vee)$$

$$\langle \equiv \rangle (\text{false} \vee p \wedge q) \wedge (\neg q \vee r) \quad \text{Law of Contradiction}$$

$$\langle \equiv \rangle p \wedge q \wedge (\neg q \vee r) \quad \text{Simplification (3.21)}$$

$$\langle \equiv \rangle p \wedge q \wedge \neg q \vee p \wedge q \wedge r \quad \text{Distributive Law } (\wedge \text{ over } \vee)$$

$$\langle \equiv \rangle p \wedge \text{false} \vee p \wedge q \wedge r \quad \text{Law of Contradiction}$$

$$\langle \equiv \rangle \text{false} \vee p \wedge q \wedge r \quad \text{Simplification (3.20)}$$

$$\langle \equiv \rangle p \wedge q \wedge r \quad \text{Simplification (3.21)}$$