

COMP 681 Formal Methods Spring 2009 Assignment 1—Solutions

Due in class, Tuesday, January 27

Total points: 23

1 (6 pts.). Encode the following into the language of propositional logic. Use the indicated schematic letters for the indicated prime propositions.

a (3 pts.). *The committee meets in the auditorium unless it is occupied, in which case the meeting is in the cafeteria if it's before 3:00 PM or in the conference room (if it's after 3:00).*

p = The committee meets in the auditorium.

q = The auditorium is occupied.

r = The committee meets in the cafeteria.

s = It is before 3:00 PM.

t = The committee meets in the conference room.

u = It is after 3:00 PM.

Answer

$(\neg q \Rightarrow p) \wedge (q \Rightarrow (s \Rightarrow r) \wedge (u \Rightarrow t))$ Note that $u \langle \equiv \rangle \neg s$.

b (3 pts.). *The discussion will cover how cars work as well as the history of the auto industry as long as there is interest, but it will not be concerned with the role of unions unless the speaker brings up this topic.*

p = The discussion will cover how cars work.

q = The discussion will cover the history of the auto industry.

r = There is interest in the history of the auto industry.

s = The discussion will be concerned with the role of unions in the history of the auto industry.

t = The speaker brings up the role of unions in the history of the auto industry.

Answer

$(q \Rightarrow (r \Rightarrow q) \wedge (s \Rightarrow t))$ Note that $s \Rightarrow t \langle \equiv \rangle \neg t \Rightarrow \neg s$.

2 (8 pts.). Encode the following into the language of propositional logic. You must identify prime propositions and associate schematic letters with them.

- a** (4 pts.). *Although the economy improves when government spending increases, the governor will not sign the bill unless it includes a provision for increased income tax, which is something the Republicans oppose when the economy is stagnant.*

Answer

Let

p = *The economy improves.*

q = *Government spending increases.*

r = *The governor signs the bill.*

s = *The bill includes a provision for increased income tax.*

t = *The Republicans oppose increased income tax.*

u = *The economy is stagnant.*

The encoding, then, is

$(q \Rightarrow p) \wedge (r \Rightarrow s) \wedge (u \Rightarrow t)$ Note that $r \Rightarrow s \langle \equiv \rangle \neg s \Rightarrow \neg r$.

- b** (4 pts.) *Bill, Ed, and John are all taking COMP 618, but only Ed is taking COMP 619, and the other two will take COMP 625 only if Ed does (which is unlikely).*

Answer

Let

p = *Bill is taking COMP 618.*

q = *Ed is taking COMP 618.*

r = *John is taking COMP 618.*

s = *Bill is taking COMP 619.*

t = *Ed is taking COMP 619.*

u = *John is taking COMP 619.*

v = *Bill will take COMP 625.*

w = *Ed will take COMP 625.*

x = *John will take COMP 625.*

y = *It is unlikely that Ed will take COMP 625.*

The encoding, then, is

$p \wedge q \wedge r \wedge \neg s \wedge t \wedge \neg u \wedge (v \wedge x \Rightarrow w) \wedge y$

3 (5 pts.). Consider the following propositional logic wff.

$$p \wedge \neg q \vee \neg r \wedge s \Rightarrow p \vee q \vee \neg(r \wedge s) \Rightarrow (s \Rightarrow r)$$

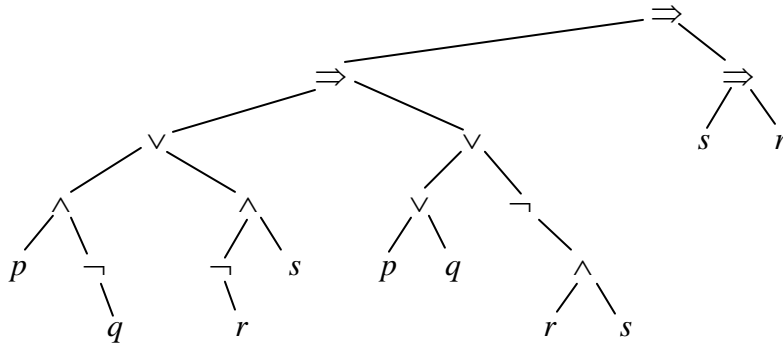
a (2 pts.). Give the fully parenthesized version of this wff.

Answer

$$(((p \wedge (\neg q)) \vee ((\neg r) \wedge s)) \Rightarrow ((p \vee q) \vee \neg(r \wedge s))) \Rightarrow (s \Rightarrow r)$$

b (3 pts.). Draw the parse tree for it.

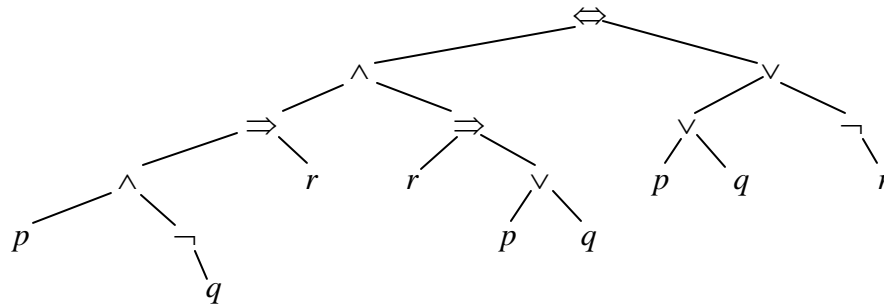
Answer



4 (4 pts.). Draw the parse tree for the following formula then give the truth table for it.

$$(p \wedge \neg q \Rightarrow r) \wedge (r \Rightarrow p \vee q) \Leftrightarrow p \vee q \vee \neg r$$

Answer



p	q	r	$\neg q$	$\neg r$	$p \wedge \neg q$	$p \wedge \neg q \Rightarrow r$ (1)	$p \vee q$	$r \Rightarrow p \vee q$ (2)	$(1) \wedge (2)$ (3)	$p \vee q \vee \neg r$ (4)	$(3) \Leftrightarrow (4)$
T	T	T	F	F	F	T	T	T	T	T	T
T	T	F	F	T	F	T	T	T	T	T	T
T	F	T	T	F	T	T	T	T	T	T	T
T	F	F	T	T	T	F	T	T	F	T	F
F	T	T	F	F	F	T	T	T	T	T	T
F	T	F	F	T	F	T	T	T	T	T	T
F	F	T	T	F	F	T	F	F	F	F	T
F	F	F	T	T	F	T	F	T	T	T	T