

1. Encode the following into the language of predicate logic with the indicated types and function symbols.

Vehicles with axle weight exceeding 18000 pounds travel at most 60 mph when crossing a bridge whose span exceeds 100 yards unless they have a permit dated after 12-31-2006.

Use the types *Vehicle*, *Bridge*, and *Permit* (with the obvious meanings), and use the following function symbols.

axle_weight(x): the axle weight of *x* in pounds

span(x): the span of *x* in yards

date(x): the date *x* was issued

speed_on(x, y): the speed in mph at which *x* travels when on *y*

Assume that dates have the format MM-DD-YYYY and that the infix binary predicate *>* is overloaded so that its arguments may be numbers or dates. You may use the usual infix binary predicates for the order relation on numbers.

Answer

$$\forall x : Vehicle, y : Bridge \bullet axle_weight(x) > 18000 \wedge span(y) > 100 \Rightarrow ((\neg \exists z : Permit \bullet date(z) > 12-31-2006) \Rightarrow speed_on(x, y) \leq 60)$$

2. Prove that the following inference is valid.

$$\begin{array}{l} \forall x \bullet (\exists y \bullet p(x,y)) \Rightarrow q(x) \\ \exists x,y \bullet p(x,y) \\ \hline \exists x \bullet q(x) \end{array}$$

Answer

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|----|---|---|
| 1. | $\forall x \bullet (\exists y \bullet p(x,y)) \Rightarrow q(x)$ | premise |
| 2. | $\exists x,y \bullet p(x,y)$ | premise |
| 3. | $x' \quad \exists y \bullet p(x',y)$ | assumption (2, \exists_E) |
| 4. | $(\exists y \bullet p(x',y)) \Rightarrow q(x')$ | from 1, \forall_E |
| 5. | $q(x')$ | from 4 and 3, modus ponens (\Rightarrow_E) |
| 6. | $\exists x \bullet q(x)$ | from 5, \exists_I |
| 7. | $\exists x \bullet q(x)$ | from 3-6, $\exists_E (x')$ |