

# COMP 681 Formal Methods Spring 2009 Quiz 2—Solutions

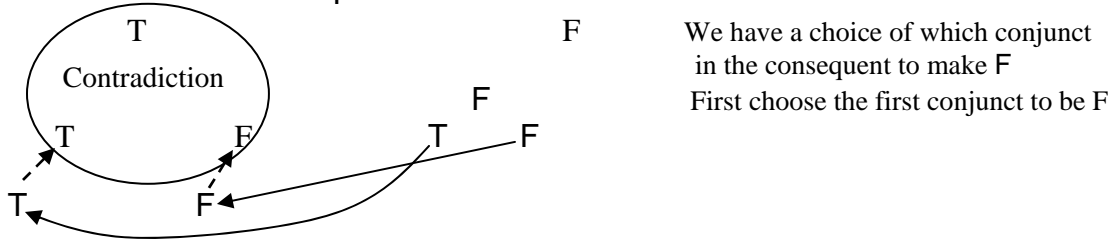
The questions are equally weighted

1. Use the shorter truth table method to show that the following is a tautology. **Hint:** You have to follow out two choices, but you handle them in the same way.

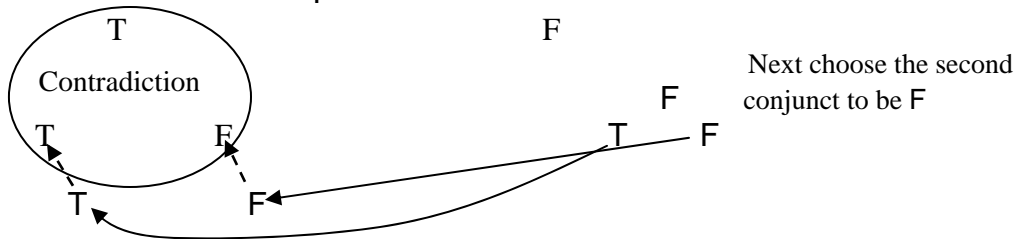
$$p \vee q \Rightarrow r \wedge s \Rightarrow (p \Rightarrow r) \wedge (q \Rightarrow s)$$

**Answer**

$$p \vee q \Rightarrow r \wedge s \Rightarrow (p \Rightarrow r) \wedge (q \Rightarrow s)$$



$$p \vee q \Rightarrow r \wedge s \Rightarrow (p \Rightarrow r) \wedge (q \Rightarrow s)$$



2. Convert the following to a logically equivalent wff in CNF then convert the result into a logically equivalent conjunction of implications without negation. Show and justify your steps in the conversion to CNF, but you needn't show your work in converting to the conjunction of implications (since you can just eye-ball it).

$$p \vee q \Rightarrow r \wedge s$$

**Answer**

$$p \vee q \Rightarrow r \wedge s$$

Step 1 does not apply.

Step 2

$$\langle \equiv \rangle \neg(p \vee q) \vee r \wedge s$$

Step 3

$$\langle \equiv \rangle \neg p \wedge \neg q \vee r \wedge s$$

Step 4

$$\langle \equiv \rangle (\neg p \wedge \neg q \vee r) \wedge (\neg p \wedge \neg q \vee s)$$

$$\langle \equiv \rangle (\neg p \vee r) \wedge (\neg q \vee r) \wedge (\neg p \vee s) \wedge (\neg q \vee s)$$

By four applications of the Law of Implication (or just by inspection) this is logically equivalent to

$$(p \Rightarrow r) \wedge (q \Rightarrow r) \wedge (p \Rightarrow s) \wedge (q \Rightarrow s)$$