COMP 681 Formal Methods Spring 2009 Exam 1—<u>Solutions</u>

30 points total. All problems are worth the same number of points.

1. Encode the following into the language of propositional logic:

When Fred is at home, he uses his laptop as long as it's not night, but, while at work, he never uses it.

Let

p = *Fred is at home.*
q = *Fred uses his laptop.*
r = *It is night.*
s = *Fred is at work*

Answer

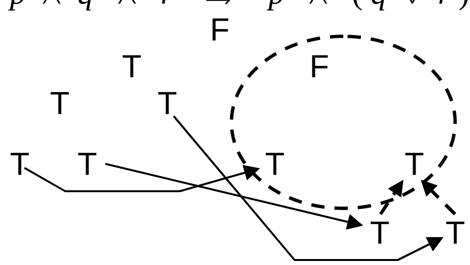
$(p \Rightarrow (\neg r \Rightarrow q)) \wedge (s \Rightarrow \neg q)$ Equivalently: $(p \wedge \neg r \Rightarrow q) \wedge (s \Rightarrow \neg q)$

2. Use the shorter truth table method to show

$p \wedge q \wedge r \equiv> p \wedge (q \vee r)$

Answer

$p \wedge q \wedge r \Rightarrow p \wedge (q \vee r)$



3. Use transformational proof to show the following. Justify each step, but you need not show the substitution used to derive the appropriate instance of the laws. You may assume implicit associativity and commutativity of $\wedge$ and $\vee$, and you may use generalized forms of the laws.

$(p \Rightarrow q) \wedge (r \vee \neg p) <\equiv> p \Rightarrow q \wedge r$

Answer

$(p \Rightarrow q) \wedge (r \vee \neg p)$	
$<\equiv> (\neg p \vee q) \wedge (r \vee \neg p)$	Law of Implication
$<\equiv> (\neg p \vee q) \wedge (\neg p \vee r)$	Commutativity of $\vee$
$<\equiv> \neg p \vee q \wedge r$	Distributive Law, $\vee$ over $\wedge$
$<\equiv> p \Rightarrow q \wedge r$	Distributive Law, $\vee$ over $\wedge$

4 A group of experts from different departments decides that procedure S_1 is done when P holds and that S_2 is done if Q is true but R is false.

a. Write a GCL alternation statement that captures these requirements. (The conditions won't be mutually exclusive at this point.)

Answer

if $P \rightarrow S_1$
$\square$ $Q \wedge \neg R \rightarrow S_2$
fi

b. You point out to the experts that their conditions are not mutually exclusive. They decide among themselves that S_1 should take priority S_2. Rewrite the conditions so that they are mutually exclusive and respect these priorities. (**Note**: No simplification is needed.)

Answer

S_1: No change

S_2: $\neg P \wedge Q \wedge \neg R$

So the new alternation statement (which you weren't required to give) is

if $P \rightarrow S_1$
$\square$ $\neg P \wedge Q \wedge \neg R \rightarrow S_2$
fi

5 Convert the following to CNF. Use the combinatorial techniques of Section 3.4.4 where appropriate. Note that the result after using these techniques cannot be simplified.

$(p \Rightarrow q \wedge r) \vee s$

Answer

$(p \Rightarrow q \wedge r) \vee s$
$\Leftrightarrow$ $\neg p \vee q \wedge r \vee s$ Law of Implication

This wff is in DNF, so we can now use the techniques of Section 3.4.4 to convert it to a logically equivalent wff in CNF. Note that there are 3 disjuncts. The first has 1 literal, the second 2, and the third 1. So the CNF will have $1 \times 2 \times 1 = 2$ conjuncts, each a disjunction of 3 literals:

$(\neg p \vee q \vee s) \wedge (\neg p \vee r \vee s)$