

1. Translate the following into the language of predicate logic with function symbols.

a. Any truck that weighs over 2000 lbs. pulls a 10-foot trailer.

Use the following predicates and functions:

truck(*x*): *x* is a *truck* (predicate)

trailer(*x*): *x* is a *trailer* (predicate)

pulls(*x*,*y*): *x* *pulls* *y* (predicate)

weight(*x*): the *weight* of *x* in pounds (function)

length(*x*): the *length* of *x* in feet (function)

b. Some horses are over 14 hands tall, but no donkeys are that tall.

horse(*x*): *x* is a *horse* (predicate)

donkey(*x*): *x* is a *donkey* (predicate)

height(*x*): the *height* of *x* (in hands, 1h = 4in) (function)

2. Let

$$\phi = \exists y \bullet p(x, y) \wedge \forall x \bullet q(x, y) \Rightarrow r(x)$$

$$t_1 = f(g(x), z)$$

Is t_1 free for x in ϕ ?

3. Show that the following is valid.

$$\forall x, y \bullet (\exists z \bullet x = \text{father}(z) \wedge z = \text{father}(y)) \Rightarrow x = \text{grandfather}(y)$$

$$\text{Bill} = \text{father}(\text{Fred})$$

$$\text{Fred} = \text{father}(\text{Tom})$$

$$\forall x \bullet \text{book}(x) \wedge \text{Dickens} = \text{author}(x) \Rightarrow \text{has_read}(\text{Bill}, x)$$

$$\forall x \bullet \text{book}(x) \wedge \text{Dickens} = \text{author}(x) \Rightarrow \text{has_read}(\text{grandfather}(\text{Tom}), x)$$