COMP 681 Formal Methods Spring 2008 Recitation 6—Solutions

1. Prove the following using a deductive proof. Justify each step. Show all substitutions used to derive appropriate instances of the rules.

a. $p \vee q \Rightarrow r$, $\neg q \Rightarrow \neg s$, s |- $q \wedge r$

Answer

1.	$p \vee q \Rightarrow r$	premise
2.	$\neg q \Rightarrow \neg s$	premise
3.	s	premise
4.	$\neg \neg s$	from 3, Law of Negation { p / s }
5.	$\neg \neg q$	from 2 & 4, $\Rightarrow\_E$ (Modus tollens) { p / $\neg q$, q / $\neg s$ }
6.	q	from 5, $\neg\_E$ (Double negation) { p / q }
7.	$q \vee p$	from 6, $\vee\_I$ (Addition) { p / q , q / p }
8.	$p \vee q$	from 7, Commutativity of $\vee$
9.	r	from 1 & 9, $\Rightarrow\_E$ (Modus ponens) { p / $p \vee q$, q / r }
10.	$q \wedge r$	from 6 & 9, $\wedge\_I$ (Conjunction) { p / q , q / r }

b. $p \Rightarrow (q \Rightarrow r)$, $r \Rightarrow s$, $p \wedge q$ |- $s \vee u$

Answer

1.	$p \Rightarrow (q \Rightarrow r)$	premise
2.	$r \Rightarrow s$	premise
3.	$p \wedge q$	premise
4.	p	from 3, $\wedge\_E$ (Simplification)
5.	$q \Rightarrow r$	from 1 & 4, $\Rightarrow\_E$ (Modus ponens) { q / $q \Rightarrow r$ }
6.	$q \wedge p$	from 1, Commutativity of $\vee$
7.	q	from 6, $\wedge\_E$ (Simplification) { p / q , q / p }
8.	r	from 5 & 7, $\Rightarrow\_E$ (Modus ponens) { p / q , q / r }
9.	s	from 2 & 8, $\Rightarrow\_E$ (Modus ponens) { p / r , q / s }
10.	$s \vee u$	from 9, $\vee\_I$ (Addition) { p / s , q / u }

2. Prove the following using a deductive proof. Justify each step, but you need not show the substitution used to derive the appropriate instance of the laws. You may use commuted and generalized versions of the rules that involve $\wedge$ and $\vee$.

$p \wedge \neg q \wedge r$, $q \vee s \vee t$, $s \Rightarrow u$, $t \Rightarrow v$ |- $v \vee u$

Answer

1.	$p \wedge \neg q \wedge r$	premise
2.	$q \vee s \vee t$	premise
3.	$s \Rightarrow u$,	premise
4.	$t \Rightarrow v$	premise
5.	$\neg q$	from 1, $\wedge$_*E* (Simplification, generalized)
6.	$s \vee t$	from 2 & 5, $\vee$_*E* (Disjunctive Syllogism, generalized)
7.	$v \vee u$	from 6, 3, & 4, Constructive Dilemma (commuted)

3. Use conditional proof to prove the following.

$p \wedge q \Rightarrow r \vee s$, $p \vee t$, $\neg t \wedge \neg s$ |- $q \Rightarrow r$

Answer

1.	$p \wedge q \Rightarrow r \vee s$	premise
2.	$p \vee t$	premise
3.	$\neg t \wedge \neg s$	premise
4.	p	from 1, $\wedge$_*E* (Simplification)
5.	q	assumption
6.	$p \wedge q$	from 4 & 5, $\wedge$_*I* (Conjunction)
7.	$r \vee s$	from 1 & 6, $\Rightarrow$_*E* (Modus ponens)
8.	$\neg s$	from 3, $\wedge$_*E* (Simplification)
9.	r	from 7 & 8, $\vee$_*E* (Disjunctive syllogism)
10.	$q \Rightarrow r$	from 5-9, $\Rightarrow$_*I*

4. Use indirect proof to prove the following.

$p \vee \neg q$, $r \Rightarrow p \vee q$, r |- p

Answer

1.	$p \vee \neg q$	premise
2.	$r \Rightarrow p \vee q$	premise
3.	r	premise
4.	$p \vee q$	from 2 & 3, $\Rightarrow$_*E* (Modus ponens)
5.	$\neg p$	assumption
6.	$\neg q$	from 1 & 5, $\vee$_*E* (Disjunctive syllogism)
7.	q	from 4 & 5, $\vee$_*E* (Disjunctive syllogism)
8.	$\neg \neg p$	from 5-7, $\neg$_*I* (*reductio ad absurdum*) (6 vs. 7)
9.	p	from 8, Double negation