## COMP 681 Formal Methods Spring 2008 Recitation 5—Solutions

**1**. Consider the following wff:

$p \Rightarrow \neg ( q \Rightarrow r \vee s )$

**a**. Convert this to CNF.

**Answer**

$p \Rightarrow \neg ( q \Rightarrow r \vee s )$

Step 1—Doesn't apply

Step 2

$\langle\equiv\rangle \ \neg p \vee \neg ( q \Rightarrow r \vee s )$ Law of Implication

$\langle\equiv\rangle \ \neg p \vee \neg ( \neg q \vee r \vee s )$ Law of Implication

Step 3

$\langle\equiv\rangle \ \neg p \ \vee \ \neg \neg q \wedge \neg r \wedge \neg s$ De Morgan's 2nd Law

$\langle\equiv\rangle \ \neg p \ \vee \ q \wedge \neg r \wedge \neg s$ Law of Negation

Step 4

$\langle\equiv\rangle \ ( \neg p \vee q ) \ \wedge \ ( \neg p \vee \neg r ) \ \wedge \ ( \neg p \vee \neg s )$ Distributive Law ($\vee$ over $\wedge$)

**b**. Convert the result of a to the normal form with implications but no negations.

**Answer**

We convert the four disjunctions separately, and we get the final result by the substitutivity of logical equivalents.

$\neg p \vee q$

$\langle\equiv\rangle \ p \Rightarrow q$ Law of Implication

$\neg p \vee \ \neg r$

$\langle\equiv\rangle \ \neg p \vee \neg r \vee \mathit{false}$ Simplification (3.21)

$\langle\equiv\rangle \ \neg ( p \wedge r ) \vee \mathit{false}$ De Morgan's 1st Law

$\langle\equiv\rangle \ p \wedge r \ \Rightarrow \ \mathit{false}$ Law of Implication

$\neg p \vee \ \neg s$

$\langle\equiv\rangle \ \neg p \vee \neg s \vee \mathit{false}$ Simplification (3.21)

$\langle\equiv\rangle \ \neg ( p \wedge s ) \vee \mathit{false}$ De Morgan's 1st Law

$\langle\equiv\rangle \ p \wedge s \ \Rightarrow \ \mathit{false}$ Law of Implication

The final result, then, is

$(p \Rightarrow q ) \wedge ( p \wedge r \ \Rightarrow \ \mathit{false} ) \wedge ( p \wedge s \ \Rightarrow \ \mathit{false} )$

**2**. Convert the following DNF formula to a logically equivalent formula in CNF.

$\neg p \wedge q \ \vee \ r \wedge \neg s \wedge t \wedge u$

**Answer**

This has 2 disjuncts, one with 2 and the other with 4 conjuncts.
So the result has 2×4 = 8 conjuncts, each with 2 disjuncts.
Working out the combinations:

$( \neg p \vee r ) \wedge ( \neg p \vee \neg s ) \wedge ( \neg p \vee t ) \wedge (\neg p \vee u ) \wedge ( q \vee r ) \wedge ( q \vee \neg s ) \wedge$
$( q \vee t ) \wedge ( q \vee u )$