COMP 681 Formal Methods Spring 2008 Recitation 3—Solutions

1. Prove the following using a transformational proof. Justify each step. Show all substitutions used to derive appropriate instances of the rules.

$p \Rightarrow q \wedge r \ \langle\equiv\rangle \ (p \Rightarrow q) \wedge (p \Rightarrow r)$

Answer

$p \Rightarrow q \wedge r$

$\langle\equiv\rangle \ \neg p \vee q \wedge r$ Law of Implication $\{ q / q \wedge r \}$

$\langle\equiv\rangle \ (\neg p \vee r) \wedge (\neg p \vee r)$ Distributive Law, $\vee$ over $\wedge$ $\{ p / \neg p \}$

$\langle\equiv\rangle \ (p \Rightarrow r) \wedge (\neg p \vee r)$ Law of Implication

$\langle\equiv\rangle \ (p \Rightarrow r) \wedge (p \Rightarrow r)$ Law of Implication $\{ q / r \}$

2. Prove the following using a transformational proof. Justify each step, but you need not show the substitution used to derive the appropriate instance of the laws. You may assume implicit associativity and commutativity of $\wedge$ and $\vee$, and you may use generalized forms of the laws.

$(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge p \ \langle\equiv\rangle \ p \wedge q \wedge r$

Answer

$(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge p$

$\langle\equiv\rangle \ \neg (p \vee q) \wedge (q \Rightarrow r) \wedge p$ Law of Implication

$\langle\equiv\rangle \ \neg (p \vee q) \wedge (\neg q \vee r) \wedge p$ Law of Implication

$\langle\equiv\rangle \ p \wedge \neg (p \vee q) \wedge (\neg q \vee r)$ Commutativity of $\wedge$

$\langle\equiv\rangle \ (p \wedge \neg p \ \vee \ p \wedge q) \wedge (\neg q \vee r)$ Distributive Law ($\wedge$ over $\vee$)

$\langle\equiv\rangle \ (\mathit{false} \ \vee \ p \wedge q) \wedge (\neg q \vee r)$ Law of Contradiction

$\langle\equiv\rangle \ p \wedge q \wedge (\neg q \vee r)$ Simplification (3.21)

$\langle\equiv\rangle \ p \wedge q \wedge \neg q \ \vee \ p \wedge q \wedge r$ Distributive Law ($\wedge$ over $\vee$)

$\langle\equiv\rangle \ p \wedge \mathit{false} \ \vee \ p \wedge q \wedge r$ Law of Contradiction

$\langle\equiv\rangle \ \mathit{false} \ \vee \ p \wedge q \wedge r$ Simplification (3.20)

$\langle\equiv\rangle \ p \wedge q \wedge r$ Simplification (3.21)