COMP 681 Formal Methods Spring 2008 Recitation 15—Solutions

1. Given the following De Morgan's law

$\neg (p_1 \wedge p_2) \ \langle\equiv\rangle \ \neg p_1 \vee \neg p_2$

prove the general form

$\neg (p_1 \wedge p_2 \wedge \ldots \wedge p_n) \ \langle\equiv\rangle \ \neg p_1 \vee \neg p_2 \vee \ldots \vee \neg p_n$

Answer

Induction on the number n of $\wedge$'s

Basis: $n = 1$

This is the given law.

Induction step

Assume that $\neg (p_1 \wedge \ldots \wedge p_k) \ \langle\equiv\rangle \ \neg p_1 \vee \ldots \vee \neg p_k$ (and show that $\neg (p_1 \wedge \ldots \wedge p_k \wedge p_{k+1}) \ \langle\equiv\rangle \ \neg p_1 \vee \ldots \vee \neg p_k \vee \neg p_{k+1}$).

$\neg (p_1 \wedge \ldots \wedge p_k \wedge p_{k+1})$

$\langle\equiv\rangle \ \neg ((p_1 \wedge \ldots \wedge p_k) \wedge p_{k+1})$	Associativity of $\wedge$
$\langle\equiv\rangle \ \neg (p_1 \wedge \ldots \wedge p_k) \vee \neg p_{k+1}$	De Morgan's law
$\langle\equiv\rangle \ (\neg p_1 \vee \ldots \vee \neg p_k) \vee \neg p_{k+1}$	Induction hypothesis
$\langle\equiv\rangle \ \neg p_1 \vee \ldots \vee \neg p_k \vee \neg p_{k+1}$	Associativity of $\vee$

2. Given the rule Conjunction

$p_1, p_2 \ \vert\text{-} \ p_1 \wedge p_2$

prove the general form of this rule

$p_1, p_2, \ldots, p_n \ \vert\text{-} \ p_1 \wedge p_2 \wedge \ldots \wedge p_n$

Answer

Induction on the number n of premises

Basis: $n = 2$

This is the given rule.

Induction step

Assume that $p_1, \ldots, p_k \ \vert\text{-} \ p_1 \wedge \ldots \wedge p_k$ (and show that $p_1, \ldots, p_k, p_{k+1} \ \vert\text{-} \ p_1 \wedge \ldots \wedge p_k \wedge p_{k+1}$).

1.	p_1	Premise
…		
k.	p_k	Premise
k+1.	p_{k+1}	Premise
k+2.	$p_1 \wedge \ldots \wedge p_k$	1-k, Induction hypothesis
k+3.	$p_1 \wedge \ldots \wedge p_k \wedge p_{k+1}$	k+2 & k+1, Conjunction