

COMP 681 Formal Methods Spring 2008 Recitation 12—Solutions

Prove that the following are valid inferences.

1. $\forall x \bullet p(x) \Rightarrow q(x), \neg \exists x \bullet q(x) \wedge \neg r(x), \exists x \bullet p(x) \wedge s(x) \quad |- \quad \exists x \bullet r(x) \wedge s(x)$

Answer

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|---|---|
| 1. $\forall x \bullet p(x) \Rightarrow q(x)$ | Premise |
| 2. $\neg \exists x \bullet q(x) \wedge \neg r(x)$ | Premise |
| 3. $\exists x \bullet p(x) \wedge s(x)$ | Premise |
| 4. $\neg \exists x \bullet \neg \neg (q(x) \wedge \neg r(x))$ | From 2, Law of Negation |
| 5. $\neg \exists x \bullet \neg (q(x) \Rightarrow r(x))$ | From 4, 2 nd Law of Implication |
| 6. $\forall x \bullet q(x) \Rightarrow r(x)$ | From 5, by $\neg \exists x \bullet \neg \varphi(x) \langle \equiv \rangle \forall x \bullet \varphi(x)$ |
| 7. $x' \quad p(x') \wedge s(x')$ | Assumption (3, \exists_E) |
| 8. $p(x')$ | From 7, Simplification (\wedge_E) |
| 9. $p(x') \Rightarrow q(x')$ | From 1, \forall_E |
| 10. $q(x') \Rightarrow r(x')$ | From 6, \forall_E |
| 11. $p(x') \Rightarrow r(x')$ | From 9 & 10, Hypothetical Syllogism |
| 12. $r(x')$ | From 11 & 8, Modus Ponens (\Rightarrow_E) |
| 13. $s(x')$ | From 7, Simplification (\wedge_E) |
| 14. $r(x') \wedge s(x')$ | From 12 & 13, Conjunction (\wedge_I) |
| 15. $\exists x \bullet r(x) \wedge s(x)$ | From 14, \exists_I |
| 16. $\exists x \bullet r(x) \wedge s(x)$ | From 7-15, $\exists_E(x')$ |

2. $\forall x \bullet p \Rightarrow (q(x) \Rightarrow r(x)), \forall x \bullet r(x) \Rightarrow s(x) \quad |- \quad \forall x \bullet p \Rightarrow (q(x) \Rightarrow r(x))$

Answer

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|---|---|
| 1. $\forall x \bullet p \Rightarrow (q(x) \Rightarrow r(x))$ | Premise |
| 2. $\forall x \bullet r(x) \Rightarrow s(x)$ | Premise |
| 3. $p \Rightarrow \forall x \bullet q(x) \Rightarrow r(x)$ | From 1, by $\forall x \bullet p \Rightarrow \varphi(x) \langle \equiv \rangle p \Rightarrow \forall x \bullet \varphi(x)$, where x doesn't occur free in p . |
| 4. x'' | |
| 5. p | Assumption |
| 6. $\forall x \bullet q(x) \Rightarrow r(x)$ | From 3 & 5, Modus Ponens (\Rightarrow_E) |
| 7. $q(x'') \Rightarrow r(x'')$ | From 6, \forall_E |
| 8. $r(x'') \Rightarrow s(x'')$ | From 2, \forall_E |
| 9. $q(x'') \Rightarrow s(x'')$ | From 7 & 8, Hypothetical Syllogism |
| 10. $p \Rightarrow q(x'') \Rightarrow s(x'')$ | From 5-9, Conditional Proof (\Rightarrow_I) |
| 11. $\forall x \bullet p \Rightarrow (q(x) \Rightarrow r(x))$ | From 4-10, $\forall_I(x'')$ |

Note: You can do it in one less step if you apply \forall_E directly to the first premise.