COMP 681 Formal Methods Spring 2008 Recitation 11—Solutions

1. Translate the following into the language of predicate logic. Be sure to explain the meanings of your predicate symbols.

a. *Players with contracts and registered agents attend the meeting if their agent is a veteran*.

Let

player(x) mean *x is a player*

registered(x) means *x is registered*

agent(x, y) mean *x is the agent of y*

contract(x) mean *x is a contract*

binds(x, y) mean x (a legal document) *is binding on y*

veteran(x) mean *x is a veteran*

at_meeting(x) mean *x is at the meeting*

Answer

$$\forall\, x, y \bullet player(x) \wedge agent(y, x) \wedge registered(y) \wedge (\, \exists\, z \bullet contract(z) \wedge binds(z, x)\,) \Rightarrow (\, veteran(y) \Rightarrow at\_meeting(x)\,)$$

This is logically equivalent to

$$\forall\, x, y \bullet player(x) \wedge (\, \exists\, z \bullet contract(z) \wedge binds(z, x)\,) \wedge agent(y, x) \wedge registered(y) \wedge veteran(y) \Rightarrow at\_meeting(x)$$

b. *Dogs with lice have vets who look after them and charge their owners*.

Let

dog(x) mean *x is a dog*

louse(x) mean *x is a louse*

parasitizes(x, y) mean *x parasitizes y*

vet(x) mean *x is a vet*

cares_for(x, y) mean *x cares for y*

owns(x, y) mean *x owns y*

charges(x, y) mean *x charges y*

Answer

$$\forall\, x \bullet dog(x) \wedge (\, \exists\, y \bullet louse(y) \wedge parasitizes(y, x)\,) \Rightarrow \exists\, z \bullet vet(z) \wedge cares\_for(z, x) \wedge \exists\, u \bullet owns(u, x) \wedge charges(z, u)$$

2. For each of the following formulae with typed quantifiers,

- express its meaning in English and
- translate it into a formula with untyped quantifiers.

The predicates and types have the obvious meanings except for the predicate explained in part **b**.

a. $\exists\, x : \mathit{Person}, y : \mathit{Dog} \bullet \mathit{old}(x) \wedge \mathit{ferocious}(y) \wedge \mathit{owns}(x, y)$

Answer

Meaning: *Some old people own ferocious dogs*.

Translating into a formula with untyped quantifiers:

$\exists\, x : \mathit{Person}, y : \mathit{Dog} \bullet \mathit{old}(x) \wedge \mathit{ferocious}(y) \wedge \mathit{owns}(x, y)$

$\langle\equiv\rangle\ \exists\, x : \mathit{Person}, y \bullet \mathit{dog}(y) \wedge \mathit{old}(x) \wedge \mathit{ferocious}(y) \wedge \mathit{owns}(x, y)$

$\langle\equiv\rangle\ \exists\, x , y \bullet \mathit{person}(x) \wedge \mathit{dog}(y) \wedge \mathit{old}(x) \wedge \mathit{ferocious}(y) \wedge \mathit{owns}(x, y)$

b. $\forall\, y : \mathit{Animal}, x : \mathit{Person} \bullet \mathit{owns}(x, y) \Rightarrow \exists\, z : \mathit{Dept} \bullet \mathit{licenses}(z, x, y)$

Let *licenses*(x, y, z) mean x *licenses* y *for* (i.e., to own) z

Answer

Meaning: *If a person owns an animal, then (s)he is licensed by some department to own that animal*.

Translating into a logically equivalent formula with untyped quantifiers:

$\forall\, y : \mathit{Animal}, x : \mathit{Person} \bullet \mathit{owns}(x, y) \Rightarrow \exists\, z : \mathit{Dept} \bullet \mathit{licenses}(z, x, y)$

$\langle\equiv\rangle\ \forall\, y : \mathit{Animal}, x : \mathit{Person} \bullet \mathit{owns}(x, y) \Rightarrow \exists\, z \bullet \mathit{dept}(z) \wedge \mathit{licenses}(z, x, y)$

$\langle\equiv\rangle\ \forall\, y : \mathit{Animal}, x \bullet \mathit{person}(x) \Rightarrow (\ \mathit{owns}(x, y) \Rightarrow \exists\, z \bullet \mathit{dept}(z) \wedge \mathit{licenses}(z, x, y)\ )$

$\langle\equiv\rangle\ \forall\, y, x \bullet \mathit{animal}(y) \Rightarrow (\ \mathit{person}(x) \Rightarrow (\ \mathit{owns}(x, y) \Rightarrow \exists\, z \bullet \mathit{dept}(z) \wedge \mathit{licenses}(z, x, y)\ )\ )$

This is logically equivalent to

$\forall\, y, x \bullet \mathit{animal}(y) \wedge \mathit{person}(x) \wedge \mathit{owns}(x, y) \Rightarrow \exists\, z \bullet \mathit{dept}(z) \wedge \mathit{licenses}(z, x, y)$

3. For the following formula with typed quantifiers and constraints,

- express its meaning in English,
- translate it into a formula without constraints (but with typed quantifiers), and
- translate the result into a formula without typed quantifiers.

$$\exists\, x : Horse \mid old(x) \bullet \forall\, y : Dog \mid young(y) \bullet faster(x, y)$$

Answer

Meaning: *There's an old horse that's faster than any young dog*.

Translating into a logically equivalent formula with no constraints:

$\exists\, x : Horse \mid old(x) \bullet \forall\, y : Dog \mid young(y) \bullet faster(x, y)$

$\langle\equiv\rangle\ \exists\, x : Horse \mid old(x) \bullet \forall\, y : Dog \bullet young(y) \Rightarrow faster(x, y)$

$\langle\equiv\rangle\ \exists\, x : Horse \bullet old(x) \wedge \forall\, y : Dog \bullet young(y) \Rightarrow faster(x, y)$

Translating into a logically equivalent formula without typed quantifiers:

$\exists\, x : Horse \bullet old(x) \wedge \forall\, y : Dog \bullet young(y) \Rightarrow faster(x, y)$

$\langle\equiv\rangle\ \exists\, x : Horse \bullet old(x) \wedge \forall\, y \bullet dog\,(y) \Rightarrow (\, young(y) \Rightarrow faster(x, y)\,)$

$\langle\equiv\rangle\ \exists\, x \bullet horse(x) \wedge old(x) \wedge \forall\, y \bullet dog\,(y) \Rightarrow (\, young(y) \Rightarrow faster(x, y)\,)$

This is logically equivalent to

$\exists\, x \bullet \forall\, y \bullet horse(x) \wedge old(x) \wedge dog\,(y) \wedge young(y) \Rightarrow faster(x, y)$