

1. Translate the following into English

$$\exists x \bullet \text{class}(x) \wedge \forall y \bullet \text{student}(y) \wedge \text{lazy}(y) \wedge \text{takes}(y,x) \Rightarrow \text{fails}(y,x)$$

where

$\text{class}(x)$ means x is a class.

$\text{student}(x)$ means x is a student.

$\text{lazy}(x)$ means x (a person) is lazy.

$\text{takes}(x,y)$ means x (a person) takes y (a class)

$\text{fails}(x,y)$ means x (a person) fails y (a class).

Answer

The following are two of many possibilities.

There is a class that is failed by all lazy students who take it.

In some classes, all lazy students fail.

2. Encode the following into the language of predicate logic.

Every customer receives a gift when he registers.

Let

$\text{customer}(x)$ mean x is a customer.

$\text{gift}(x)$ mean x is a gift.

$\text{registers}(x)$ mean x registers.

$\text{receives}(x,y)$ mean x receives y .

Answer

$$\forall x \bullet \text{customer}(x) \Rightarrow (\text{registers}(x) \Rightarrow \exists y \bullet \text{gift}(y) \wedge \text{receives}(x,y))$$

3. What does the following abstract program specify?

$$y : [\exists z \bullet \text{age}(z,x) \wedge z \geq 21, \\ \exists z \bullet \text{parent}(z,x) \wedge (\text{sister}(y,z) \vee \exists u \bullet \text{brother}(u,z) \wedge \text{wife}(y,u))]$$

where

$\text{age}(x,y)$ means x (a natural number) is the age of y (a person).

$\text{parent}(x,y)$ means x is a parent of y .

$\text{sister}(x,y)$ means x is a sister of y .

$\text{brother}(x,y)$ means x is a brother of y .

$\text{wife}(x,y)$ means x is the wife y .

Answer

As long as x is at least 21 years old, make y equal to an aunt of x .

4. Write an abstract program that specifies the following. As long as the sum of y and z is no greater than x , increment both y and z by x . (x is read-only while y and z are read-write.)

Answer

$$y, z : [y + z \leq x, y = y_0 + x \wedge z = z_0 + x]$$