COMP 681 Formal Methods Spring 2008 Assignment 6

Due Monday, April 28

1. Translate the following into the language of predicate logic using the indicated function symbols.

a. *People who work in the same office have the same boss.*

Use the function symbols *office*(_) and *boss*(_), where

$office(x) = y$ means y is the office in which x works, and

$boss(x) = y$ means y is x's boss.

c. *No student at A&T has a higher GPA than John's roommate.*

Use the constant symbols *A&T* and *John* and the function symbols *college_of*(_), *gpa*(_), and *roommate*(_), where

$college\_of(x) = y$ means y is the college x attends,

$gpa(x) = y$ means y is x's GPA, and

$roommate(x) = y$ means y is x's roommate.

2. Let

$\varphi = \forall x \bullet \exists z \bullet p(z, x, h(y)) \Rightarrow q(x, y)$

$t_1 = f(g(u), v)$

$t_2 = f(y, g(z))$

a. Is t_1 free for y in φ?

b. Is t_2 free for y in φ?

Explain your answers!

3. Show that the following argument is valid.

$\forall x \bullet (\exists y \bullet p(x, y) \wedge q(y)) \Rightarrow f(x) = g(x)$

$p(A, B)$

$q(B)$

$\forall x \bullet p(x, B) \Rightarrow h(f(x), g(A)) = D$

$h(g(A), g(A)) = D$

4. Prove the following by simple (weak) induction.

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

5. Prove the following by strong induction.

The largest element in a subtree of a heap is at the root of the subtree. (You are given only the heap condition. "Subtree" here is taken in the sense of a tree that is a [not necessarily proper] part of a tree. [Thus, for example, a heap is a subtree of itself.])

6. You are given the following valid predicate-logic formula:

$\forall x \bullet \varphi(x) \wedge \psi(x) \ \langle\equiv\rangle \ (\forall x \bullet \varphi(x)) \wedge (\forall x \bullet \psi(x))$

By induction, prove that the generalization of this is valid, that is, prove

$\forall x \bullet \varphi_0(x) \wedge \varphi_1(x) \wedge \ldots \wedge \varphi_n(x) \ \langle\equiv\rangle \ (\forall x \bullet \varphi_0(x)) \wedge (\forall x \bullet \varphi_1(x)) \wedge .. \wedge (\forall x \bullet \varphi_n(x))$

7. You are given the following version of the rule Constructive Dilemma:

$p \Rightarrow q$
$r \Rightarrow s$
$p \vee r$

$q \vee s$

By induction, prove the generalization of this:

$p_1 \Rightarrow q_1$
$p_2 \Rightarrow q_2$
…
$p_n \Rightarrow q_n$
$p_1 \vee p_2 \vee \ldots \vee p_n$

$q_1 \vee q_2 \vee \ldots \vee q_n$