

COMP 681 Formal Methods Spring 2008 Assignment 4

1. Prove the following using deductive proofs. Justify each step. Show all substitutions used to derive appropriate instances of the rules.

a. $p \wedge q \Rightarrow r, p \vee \neg s, s \Rightarrow q, t \wedge s \mid\!-\! r$

b. $p \Rightarrow t \wedge q, r \vee p \vee \neg s, s \wedge \neg r \mid\!-\! s \wedge t$

2. Prove the following using deductive proofs. Justify each step, but you need not show the substitution used to derive the appropriate instance of the laws. You may use commuted and generalized versions of the rules that involve \wedge and \vee .

a. $p \wedge q \vee r \wedge s, r \Rightarrow w \vee t, t \Rightarrow u, \neg(u \vee w), p \Rightarrow (q \Rightarrow v) \mid\!-\! v \wedge \neg(u \vee r)$

b. $p \wedge q \Rightarrow r, s \wedge t \Rightarrow u, p \vee \neg v, w \Rightarrow q \wedge s, w \vee y \Rightarrow t, v \wedge w \mid\!-\! r \wedge u$

3. Use conditional proof to prove the following.

a. $\neg p \vee (q \Rightarrow t \wedge u), p \wedge q \Rightarrow v \vee w, \neg v, w \Rightarrow s \mid\!-\! p \Rightarrow (q \wedge r \Rightarrow s \wedge t)$

b. $p \Rightarrow u, u \Rightarrow q \wedge w, \neg r \vee \neg w \vee s, q \wedge r \Rightarrow t \mid\!-\! p \Rightarrow q \wedge (r \Rightarrow s \wedge t)$

4. Use indirect proof to prove the following.

$$s \Rightarrow (r \Rightarrow p), s \Rightarrow (\neg r \Rightarrow p), \neg q \vee s, q \vee s \mid\!-\! p$$

5. For each of the following natural-language arguments, do the following.

- Encode the argument as a propositional-logic inference. Be sure to define the meanings of the prime propositions that you use.
- Use the shorter truth table method to show that the argument is valid.
- Prove that the argument is valid using inference rules and logical equivalences.

a. *If the red button is clicked, the screen shrinks but the image's size doesn't change. The text disappears whenever the screen shrinks, but the submit button is still visible. So, when the red button is clicked, the image doesn't change in size and the submit button remains visible.*

b. *The crosswalk is closed when the train is in the station. The stationmaster is in his office whenever the crosswalk is closed or a train is being monitored. So, the stationmaster is in his office because the gates are closed, and, when the gates are closed, the train is in the station.*