

COMP 681 Formal Methods Spring 2008 Assignment 2

1. Use the shorter truth table method to determine whether the following are tautologies.

a. $(p \vee q \Rightarrow r \wedge s) \Rightarrow (p \Rightarrow r \vee s) \wedge (q \Rightarrow r)$

b. $(p \vee q \Rightarrow r \wedge s) \Rightarrow (p \vee \neg r \Rightarrow \neg q \wedge s)$

c. $p \wedge q \Rightarrow r \vee s \Leftrightarrow p \wedge \neg r \Rightarrow \neg q \vee s$

d. $p \wedge q \vee p \wedge \neg q \wedge r \vee p \wedge \neg q \wedge \neg r \vee \neg p$

2. Use the shorter truth table method to determine whether the following are contradictions.

a. $(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge (r \Rightarrow \neg p) \wedge p$

b. $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$

3. Prove the following using a transformational proof. Justify each step. Show all substitutions used to derive appropriate instances of the rules.

a. $\neg p \wedge \neg q \Rightarrow \neg r \langle \equiv \rangle r \Rightarrow p \vee q$

b. $p \Rightarrow (q \Rightarrow r) \langle \equiv \rangle p \wedge q \Rightarrow r$

c. $p \Rightarrow q \langle \equiv \rangle \neg(p \wedge \neg q)$

4. Prove the following using a transformational proof. Justify each step, but you need not show the substitution used to derive the appropriate instance of the laws. You may assume implicit associativity and commutativity of \wedge and \vee , and you may use generalized forms of the laws.

a. $(p \Rightarrow q) \Rightarrow (r \Rightarrow s) \langle \equiv \rangle r \Rightarrow (\neg p \Rightarrow s) \wedge (q \Rightarrow s)$

b. $p \wedge q \wedge \neg r \Rightarrow r \vee \neg(p \wedge q) \langle \equiv \rangle p \Rightarrow (q \Rightarrow r)$