

## COMP 681 Formal Methods Spring 2008 Assignment 1—Solutions

1 (6 pts.). Encode the following into the language of propositional logic. Use the indicated schematic letters for the indicated prime propositions.

- a (3 pts.). *COMP 618 meets Mondays, Wednesdays, and Fridays except when all classes are cancelled or Prof. Smith, who teaches this class, is out of town.*

$p$  = *COMP 618 meets Mondays.*

$q$  = *COMP 618 meets Wednesdays.*

$m$  = *COMP 618 meets Fridays.*

$r$  = *All classes are cancelled.*

$s$  = *Prof. Smith is out of town.*

$t$  = *Prof. Smith teaches COMP 618.*

**Answer**

$$(\neg(r \vee s) \Rightarrow p \wedge q \wedge m) \wedge t$$

- b (3 pts.). *Besides manufacturing paper, Acme, Inc. is a supplier of cellulose, which is used in the production of cellophane and rayon, as well as (given there's a demand) garden mulch.*

$p$  = *Acme, Inc. manufactures paper.*

$q$  = *Acme, Inc. supplies cellulose.*

$r$  = *Cellulose is used in the production of cellophane.*

$s$  = *Cellulose is used in the production of rayon.*

$t$  = *There is a demand for garden mulch.*

$u$  = *Acme, Inc. is a supplier of garden mulch.*

**Answer**

$$p \wedge q \wedge r \wedge s \wedge (t \Rightarrow u)$$

2 (8 pts.). Encode the following into the language of propositional logic. You must identify prime propositions and associate schematic letters with them.

- a (4 pts.). *The shipping department will increase rates if sales increase while costs hold steady even though, when sales increase, wages do as well, and increased wages lead to decreased profits.*

**Answer**

Let

$p$  = *The shipping department increases rates.*

$q$  = *Sales increase.*

$r$  = *Costs hold steady.*

$s$  = *Wages increase.*

$t$  = *Profits decrease.*

Then

$$(q \wedge r \Rightarrow p) \wedge (q \Rightarrow s) \wedge (s \Rightarrow t)$$

**b** (4 pts.). *Unlike Fred, Bill passed COMP 618, but he, like ~~Bill~~ Fred (correction), will take COMP 619 only if he also takes COMP 620, and that requires that he took COMP 520.*

**Answer**

Let

- $p = \text{Bill passed COMP 618.}$
- $q = \text{Fred passed COMP 618.}$
- $r = \text{Bill takes COMP 619.}$
- $s = \text{Fred takes COMP 619.}$
- $t = \text{Bill takes COMP 620.}$
- $u = \text{Fred takes COMP 620.}$
- $v = \text{Bill took COMP 520.}$
- $w = \text{Fred took COMP 520.}$

Then

$$p \wedge \neg q \wedge (r \Rightarrow t) \wedge (s \Rightarrow u) \wedge (t \Rightarrow v) \wedge (u \Rightarrow w)$$

**3** (5 pts.). Consider the following propositional logic wff.

$$\neg p \wedge q \vee r \Rightarrow (q \Rightarrow \neg r) \wedge s \Rightarrow \neg r \vee \neg (s \vee t)$$

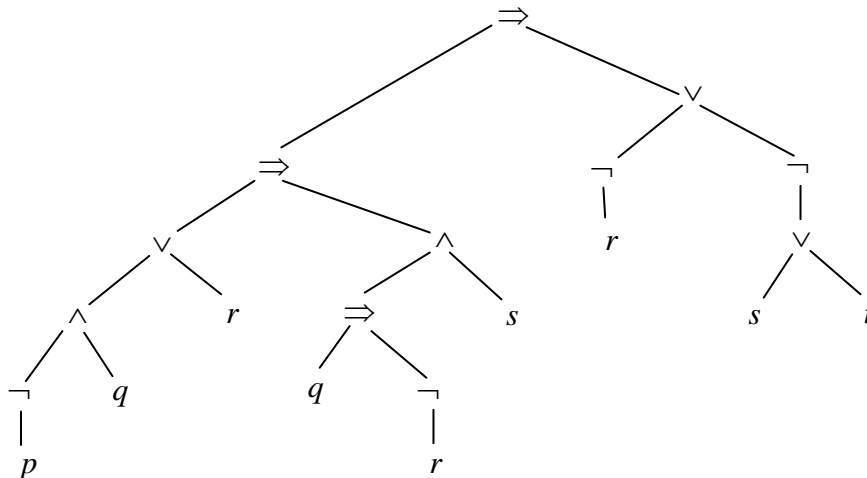
**a** (2 pts.). Give the fully parenthesized version of this wff.

**Answer**

$$(((\neg p) \wedge q) \vee r) \Rightarrow ((q \Rightarrow (\neg r)) \wedge s) \Rightarrow ((\neg r) \vee (\neg (s \vee t)))$$

**b** (3 pts.). Draw the parse tree for it.

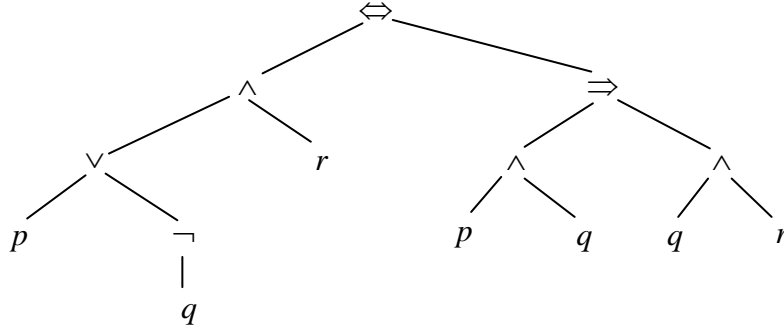
**Answer**



4 (8 pts.). For each of the following, draw the parse tree for the following formula then give the truth table for it.

a (4 pts.).  $(p \vee \neg q) \wedge r \Leftrightarrow p \wedge q \Rightarrow q \wedge r$

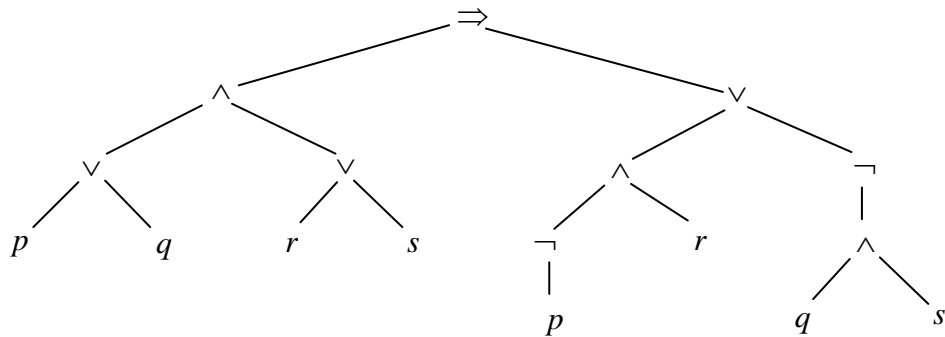
**Answer**



$p$	$q$	$r$	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \wedge r$ (1)	$p \wedge q$	$q \wedge r$	$p \wedge q \Rightarrow q \wedge r$ (2)	$(1) \Leftrightarrow (2)$
T	T	T	F	T	T	T	T	T	T
T	T	F	F	T	F	T	F	F	T
T	F	T	T	T	T	F	F	T	T
T	F	F	T	T	F	F	F	T	F
F	T	T	F	F	F	F	T	T	F
F	T	F	F	F	F	F	F	T	F
F	F	T	T	T	T	F	F	T	T
F	F	F	T	T	F	F	F	T	F

b (4 pts.).  $(p \vee q) \wedge (r \vee s) \Rightarrow \neg p \wedge r \vee \neg(q \wedge s)$

**Answer**



$p$	$q$	$r$	$s$	$\neg p$	$p \vee q$	$r \vee s$	$(p \vee q) \wedge (r \vee s)$ (1)	$\neg p \wedge r$ (2)	$q \wedge s$	$\neg(q \wedge s)$ (3)	$(2) \vee (3)$ (4)	$(1) \Rightarrow (4)$
T	T	T	T	F	T	T	T	F	T	F	F	F
T	T	T	F	F	T	T	T	F	F	T	T	T
T	T	F	T	F	T	T	T	F	T	F	F	F
T	T	F	F	F	T	F	F	F	F	T	T	T
T	F	T	T	F	T	T	T	F	F	T	T	T
T	F	T	F	F	T	T	T	F	F	T	T	T
T	F	F	T	F	T	T	T	F	F	T	T	T
T	F	F	F	F	T	F	F	F	F	T	T	T
F	T	T	T	T	T	T	T	T	T	F	T	T
F	T	T	F	T	T	T	T	T	F	T	T	T
F	T	F	T	T	T	T	T	F	T	F	F	F
F	T	F	F	T	T	F	F	F	F	T	T	T
F	F	T	T	T	F	T	F	T	F	T	T	T
F	F	T	F	T	F	T	F	T	F	T	T	T
F	F	F	T	T	F	T	F	F	F	T	T	T
F	F	F	F	T	F	F	F	F	F	T	T	T