COMP 681 Formal Methods Spring 2008 Quiz 3

1. Show that the following is valid.

$\forall x \bullet (\exists y \bullet p(y, x)) \Rightarrow f(x) = g(x)$
$p(a, b)$
$q(f(b))$

$q(g(b))$

Answer

1. $\forall x \bullet (\exists y \bullet p(y, x)) \Rightarrow f(x) = g(x)$ Premise
2. $p(a, b)$ Premise
3. $q(f(b))$ Premise
4. $(\exists y \bullet p(y, b)) \Rightarrow f(b) = g(b)$ from 1, ∀_E (b/x)
5. $\exists y \bullet p(y,b)$ from 2, ∃_I (introducing var. y for const. b)
6. $f(b) = g(b)$ from 4 & 5, ⇒_E
7. $q(g(b))$ from 6 & 3, =_E

2. You are given the following version of the rule Disjunctive Syllogism:

$p \vee q$
$\neg p$
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q

By induction, prove the generalization of this:

$p_1 \vee p_2 \vee \ldots \vee p_n \vee q$
$\neg p_1$
$\neg p_2$
$\ldots$
$\neg p_n$
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q

Answer (see next page)

Answer

By weak induction on the number n of $\vee$'s in the first premise (= the number of premises of the form $\neg p_i$)

Basis: $n = 1$

This is the given rule.

Induction step

Assume that, for $k \geq 1$, we have

$$p_1 \vee p_2 \vee \ldots \vee p_k \vee q,\ \neg p_1,\ \neg p_2,\ \ldots,\ \neg p_k\ \vdash\ q$$

(and show that

$$p_1 \vee p_2 \vee \ldots \vee p_k \vee p_{k+1} \vee q,\ \neg p_1,\ \neg p_2,\ \ldots,\ \neg p_k,\ \neg p_{k+1}\ \vdash\ q)$$

1.	$p_1 \vee p_2 \vee \ldots \vee p_k \vee p_{k+1} \vee q$	Premise
2.	$\neg p_1$	Premise
3.	$\neg p_2$	Premise
…		
k+1.	$\neg p_k$	Premise
k+2.	$\neg p_{k+1}$	Premise
k+3.	$p_{k+1} \vee p_1 \vee p_2 \vee \ldots \vee p_k \vee q$	from 1, Commutativity of $\vee$
k+4.	$p_{k+1} \vee (p_1 \vee p_2 \vee \ldots \vee p_k \vee q)$	from k+3, Associativity of $\vee$
k+5.	$p_1 \vee p_2 \vee \ldots \vee p_k \vee q$	from k+4 & k+2, Disjunctive Syllogism
k+6.	q	from k+5 and 2 to k+1, induction hypothesis