COMP 681 Formal Methods Spring 2008 Quiz 2—Solutions

1. Convert the following to an equivalent wff in CNF. Justify each stop. (You do not have to show the substitutions you make to apply the rules and laws.)

$(p \Rightarrow q) \vee (p \Rightarrow r \wedge s)$

Answer

$(p \Rightarrow q) \vee (p \Rightarrow r \wedge s)$

Step 1. Eliminate $\Leftrightarrow$'s

Nothing to do

Step 2. Eliminate $\Rightarrow$'s

$\langle\equiv\rangle\ (\neg p \vee q) \vee (\neg p \vee r \wedge s)$	Law of Implication (×2)
$\langle\equiv\rangle\ \neg p \vee q \vee \neg p \vee r \wedge s$	Associativity of $\vee$
$\langle\equiv\rangle\ \neg p \vee \neg p \vee q \vee r \wedge s$	Commutativity of $\vee$
$\langle\equiv\rangle\ \neg p \vee q \vee r \wedge s$	Idempotence of $\vee$

Step 3. Move $\neg$'s inward

Nothing to do

Step 4. Use distributive law

$\langle\equiv\rangle\ \underline{(\neg p \vee q \vee r) \wedge (\neg p \vee q \vee s)}$	Distributive law ($\wedge$ over $\vee$)

2. Use conditional proof to prove the following. Justify each stop. (You do not have to show the substitutions you make to apply the rules and laws.)

$p \Rightarrow s,\ s \vee t \Rightarrow r\ \vdash\ p \wedge q \Rightarrow r$

Answer

1. $p \Rightarrow s$		Premise
2. $s \vee t \Rightarrow r$		Premise
3. $p \wedge q$		Assumption
4.	p	3, $\wedge$_E (Simplification)
5.	s	1,4, $\Rightarrow$_E (modus ponens)
6.	s $\vee$ t	5, $\vee$_I (Addition)
7.	r	2, 6, $\Rightarrow$_E (modus ponens)
8. $p \wedge q \Rightarrow r$		3-7, $\Rightarrow$_I (Conditional Proof)