COMP 681 Formal Methods Spring 2008 Exam 1—<u>Solutions</u>

All questions are equally weighted.

1. Encode the following into the language of propositional logic. Define the prime propositions that you use.

If I-20 is closed, then either I-30 or I-40 is crowded; whenever I-30 is crowded, there are delays in driving to Gotham, but, when I-40 is crowded, we experience such delays only when I-50 is closed.

Answer

Let

p = *I-20 is closed*.
q = *I-30 is crowded*.
r = *I-40 is crowded*.
s = *There are delays in driving to Gotham*.
t = *I-50 is closed*.

Then

$$(p \Rightarrow q \vee r) \wedge (q \Rightarrow s) \wedge (r \Rightarrow (s \Rightarrow t))$$

2. Prove the following using a transformational proof. Justify each step, but you need not show the substitution used to derive the appropriate instance of the laws. You may assume implicit associativity and commutativity of $\wedge$ and $\vee$, and you may use generalized forms of the laws.

$$p \wedge \neg q \Rightarrow r \vee \neg s \;\langle\equiv\rangle\; \neg r \wedge s \Rightarrow (p \Rightarrow q)$$

Answer

$p \wedge \neg q \Rightarrow r \vee \neg s$	
$\langle\equiv\rangle\; \neg (r \vee \neg s) \Rightarrow \neg (p \wedge \neg q)$	Contraposition
$\langle\equiv\rangle\; \neg r \wedge \neg\neg s \Rightarrow \neg (p \wedge \neg q)$	De Morgan's 2nd Law
$\langle\equiv\rangle\; \neg r \wedge s \Rightarrow \neg (p \wedge \neg q)$	Law of Negation
$\langle\equiv\rangle\; \neg r \wedge s \Rightarrow \neg p \vee \neg\neg q$	De Morgan's 1st Law
$\langle\equiv\rangle\; \neg r \wedge s \Rightarrow \neg p \vee q$	Law of Negation
$\langle\equiv\rangle\; \neg r \wedge s \Rightarrow p \Rightarrow q$	Law of Implication

3. Convert the following into CNF. Then convert the CNF formula into the normal form with implications but no negations.

$\neg p \wedge s \vee (p \vee \neg q \Rightarrow r)$

Answer

$\neg p \wedge s \vee (p \vee \neg q \Rightarrow r)$

Step 1: Nothing to do

Step 2:

$\langle\equiv\rangle\ \neg p \wedge s \vee \neg (p \vee \neg q) \vee r$ Law of Implication

Step 3:

$\langle\equiv\rangle\ \neg p \wedge s \ \vee\ \neg p \wedge \neg\neg q \ \vee r$ De Morgan's 2^{nd} Law

$\langle\equiv\rangle\ \neg p \wedge s \ \vee\ \neg p \wedge q \ \vee r$ Law of Negation

Step 4:

$\langle\equiv\rangle\ \neg p \wedge (s \vee q) \ \vee r$ Distributive Law ($\wedge$ over $\vee$)

$\langle\equiv\rangle\ (\neg p \vee r) \wedge (s \vee q \vee r)$ Distributive Law ($\vee$ over $\wedge$)

Convert the first conjunct into the normal form with implications but no negations:

$\neg p \vee r$

$\langle\equiv\rangle\ p \Rightarrow r$ Law of Implication

Convert the second conjunct into the normal form with implications but no negations:

$s \vee q \vee r$

$\langle\equiv\rangle\ \mathit{false} \vee s \vee q \vee r$ Simplification

$\langle\equiv\rangle\ \neg\, \mathit{true} \vee s \vee q \vee r$ Since $\mathit{false} \langle\equiv\rangle \neg\, \mathit{true}$

$\langle\equiv\rangle\ \mathit{true} \ \Rightarrow\ s \vee q \vee r$ Law of Implication

Substituting into the wff in CNF:

$(p \Rightarrow r) \wedge (\mathit{true} \ \Rightarrow\ s \vee q \vee r)$

4. Translate the following into the language of predicate logic.

Inventory items consigned to a distributor are kept in warehouses while other items remain in factories.

Use the following predicates.

inventory(*x*) means *x is an inventory item.*

consigned_to(*x*, *y*) means *x is consigned to y*.

distributor(*x*) means *x is a distributor*.

in(*x*, *y*) means *x is in y*.

warehouse(*x*) means *x is a warehouse*.

factory(*x*) mans *x is a factory*.

Answer

$$\forall x \bullet \mathit{inventory}(x) \Rightarrow$$
$$((\exists y \bullet \mathit{distributor}(y) \wedge \mathit{consigned\_to}(x,y)) \Rightarrow \exists z \bullet \mathit{warehouse}(z) \wedge \mathit{in}(x,z))$$
$$\wedge (\neg (\exists y \bullet \mathit{distributor}(y) \wedge \mathit{consigned\_to}(x,y)) \Rightarrow \exists z \bullet \mathit{factory}(z) \wedge \mathit{in}(x,z))$$