

## **Project # 1**

# **Analysis of Antisymmetric Angle-ply Laminated Plates subjected to Distributed Transverse Load and Simply Supported on All Edges**

*by G Swaminathan*

*Composite Structures*

*MEEN 813*

*Instructor: Dr K N Shivakumar*

### Properties:

$$A_{16} = A_{26} = 0$$

$$B_{11} = B_{12} = B_{22} = B_{66} = 0$$

$$D_{16} = D_{26} = 0$$

### GDE:

$$A_{11}u_{,xx} + A_{66}u_{,yy} + (A_{12} + A_{66})v_{,xy} - 3B_{16}w_{,xxy} - B_{26}w_{,yyy} = 0$$

$$A_{66}v_{,xx} + A_{22}v_{,yy} + (A_{12} + A_{66})u_{,xy} - B_{16}w_{,xxx} - 3B_{26}w_{,xyy} = 0$$

$$D_{11}w_{,xxxx} + 2(D_{12} + 2D_{66})w_{,xxyy} + D_{22}w_{,yyyy} - B_{16}(3u_{,xxy} + v_{,xxx}) - B_{26}(u_{,yyy} + 3v_{,xyy}) = p(x,y)$$

### Boundary conditions:

$$@ x=0 \& a \quad w=0 \& M_x = B_{16}(u_{,y} + v_{,x}) - D_{11}w_{,xx} - D_{12}w_{,yy} = 0$$

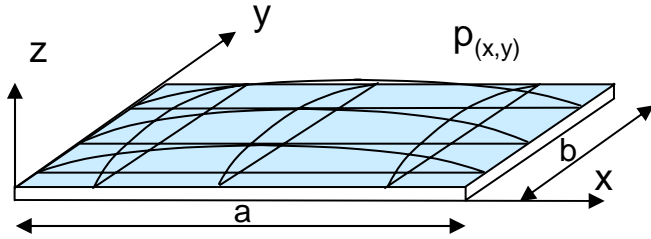
$$u=0 \& N_{xy} = A_{66}(u_{,y} + v_{,x}) - B_{16}w_{,xx} - B_{26}w_{,yy} = 0$$

$$@ y=0 \& b \quad w=0 \& M_y = B_{26}(u_{,y} + v_{,x}) - D_{12}w_{,xx} - D_{22}w_{,yy} = 0$$

$$v=0 \& N_{xy} = A_{66}(u_{,y} + v_{,x}) - B_{16}w_{,xx} - B_{26}w_{,yy} = 0$$

## Case 1

### Sinusoidal Load



$$p_{(x,y)} = q_0 \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right)$$

$$A_{mn} = \frac{T_{11}T_{23} - T_{22}T_{13}}{\Delta} q_0 \quad B_{mn} = \frac{T_{12}T_{13} - T_{11}T_{23}}{\Delta} q_0 \quad C_{mn} = \frac{T_{11}T_{22} - T_{12}^2}{\Delta} q_0 \quad \& \quad \Delta = \begin{vmatrix} T_{11} & T_{12} & T_{13} \\ T_{11} & T_{12} & T_{13} \\ T_{11} & T_{12} & T_{13} \end{vmatrix}$$

$$T_{11} = A_{11} \left(\frac{\pi}{a}\right)^2 + A_{66} \left(\frac{\pi}{b}\right)^2$$

$$T_{22} = A_{66} \left(\frac{\pi}{a}\right)^2 + A_{22} \left(\frac{\pi}{b}\right)^2$$

$$T_{12} = (A_{11} + A_{66}) \left(\frac{\pi}{a}\right) \left(\frac{\pi}{b}\right)$$

$$T_{13} = - \left[ B_{16} \left(\frac{\pi}{a}\right)^2 + 3B_{26} \left(\frac{\pi}{b}\right)^2 \right] \left(\frac{\pi}{a}\right)$$

$$T_{13} = - \left[ 3B_{16} \left(\frac{\pi}{a}\right)^2 + B_{26} \left(\frac{\pi}{b}\right)^2 \right] \left(\frac{\pi}{b}\right)$$

$$T_{33} = D_{11} \left(\frac{\pi}{a}\right)^4 + 2(D_{12} + 2D_{66}) \left(\frac{\pi}{a}\right)^2 \left(\frac{\pi}{b}\right)^2 + D_{22} \left(\frac{\pi}{b}\right)^4$$

### Displacement Functions:

$$u = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right)$$

$$v = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

where  
m=n=1

## Solutions – for square plate

**Case 1: Sinusoidal loading**

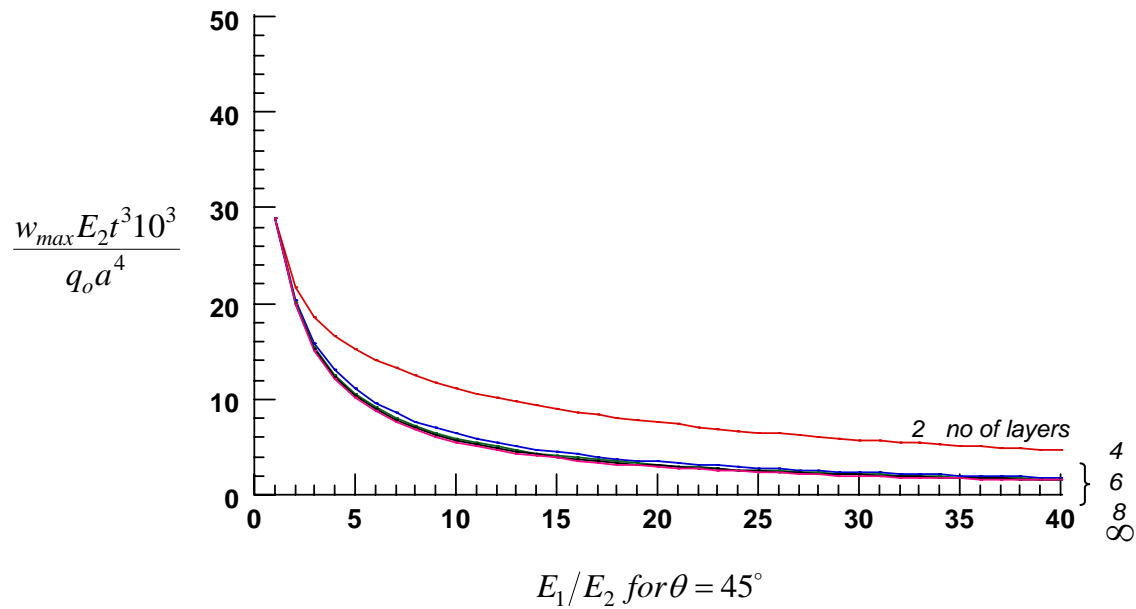
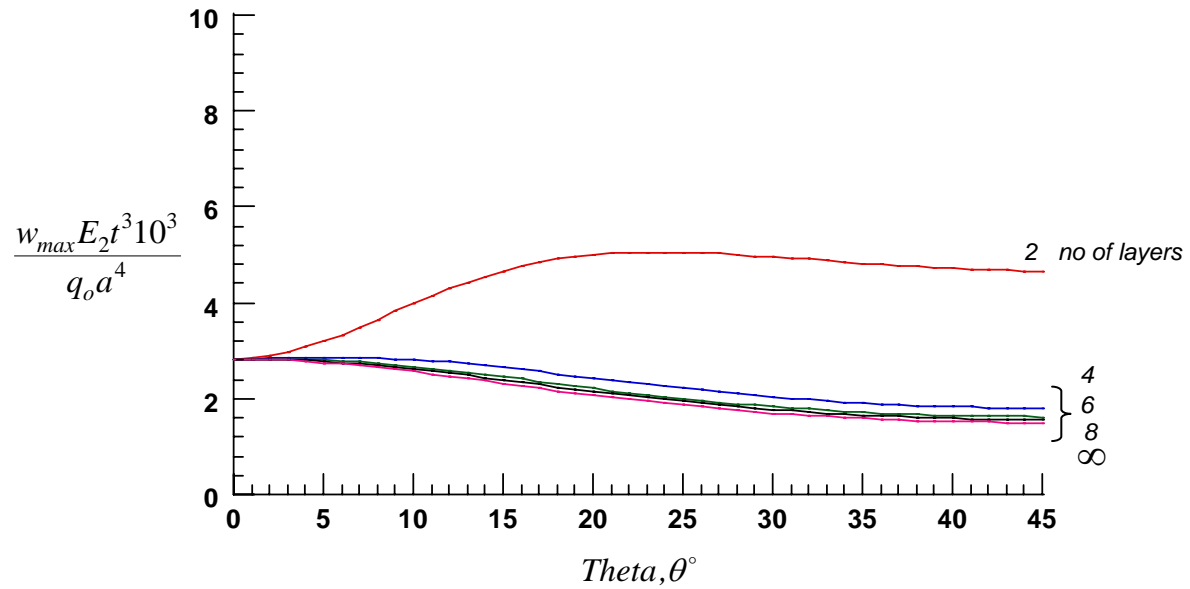
$$W_{max} = \frac{\alpha q_0 a^4}{E_2 t^3}$$

### Example 1: Graphite/epoxy laminate (same as in book)

$$\frac{E_1}{E_2} = 40 \quad \frac{G_{12}}{E_2} = 0.5 \quad E_2 = 1.0675 \text{ Msi} \quad \nu_{12} = 0.25 \quad \text{Thickness} = 0.044''$$

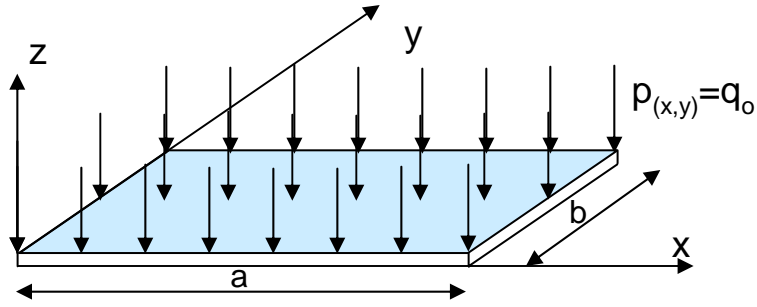
No of layers	$\alpha \times 10^3$ for various theta					$\alpha \times 10^3$ for various $E_1/E_2$ @ $\theta = 45^\circ$				
	0°	10°	20°	30°	45°	1	10	20	30	40
2	2.8278	3.9802	4.9998	4.9569	4.6673	28.873	11.175	7.574	5.770	4.667
4	2.8278	2.8129	2.4258	2.0336	1.8065	28.873	6.363	3.454	2.372	1.806
6	2.8278	2.6680	2.2147	1.8334	1.6223	28.873	5.893	3.138	2.138	1.622
8	2.8278	2.6208	2.1492	1.7772	1.5664	28.873	5.745	3.041	2.068	1.566
$\infty$	2.8278	2.5624	2.0705	1.6995	1.4999	28.873	5.564	2.924	1.982	1.499

## Case 1: Sinusoidal loading (as in book)



## Case 2

### Uniformly Distributed Load (UDL)



$$p_{(x,y)} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} P_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \quad P_{mn} = \frac{16q_0}{\pi^2 mn}$$

$$A_{mn} = \frac{T_{11}T_{23} - T_{22}T_{13}}{\Delta} P_{mn} \quad B_{mn} = \frac{T_{12}T_{13} - T_{11}T_{23}}{\Delta} P_{mn} \quad C_{mn} = \frac{T_{11}T_{22} - T_{12}^2}{\Delta} P_{mn} \quad \& \quad \Delta = \begin{vmatrix} T_{11} & T_{12} & T_{13} \\ T_{11} & T_{12} & T_{13} \\ T_{11} & T_{12} & T_{13} \end{vmatrix}$$

$$T_{11} = A_{11} \left(\frac{m\pi}{a}\right)^2 + A_{66} \left(\frac{n\pi}{b}\right)^2$$

$$T_{22} = A_{66} \left(\frac{m\pi}{a}\right)^2 + A_{22} \left(\frac{n\pi}{b}\right)^2$$

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$$T_{13} = - \left[ B_{16} \left(\frac{m\pi}{a}\right)^2 + 3B_{26} \left(\frac{n\pi}{b}\right)^2 \right] \left(\frac{m\pi}{a}\right)$$

$$T_{13} = - \left[ 3B_{16} \left(\frac{m\pi}{a}\right)^2 + B_{26} \left(\frac{n\pi}{b}\right)^2 \right] \left(\frac{n\pi}{b}\right)$$

$$T_{33} = D_{11} \left(\frac{m\pi}{a}\right)^4 + 2(D_{12} + 2D_{66}) \left(\frac{m\pi}{a}\right)^2 \left(\frac{n\pi}{b}\right)^2 + D_{22} \left(\frac{n\pi}{b}\right)^4$$

### Displacement Functions:

$$u = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right)$$

$$v = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

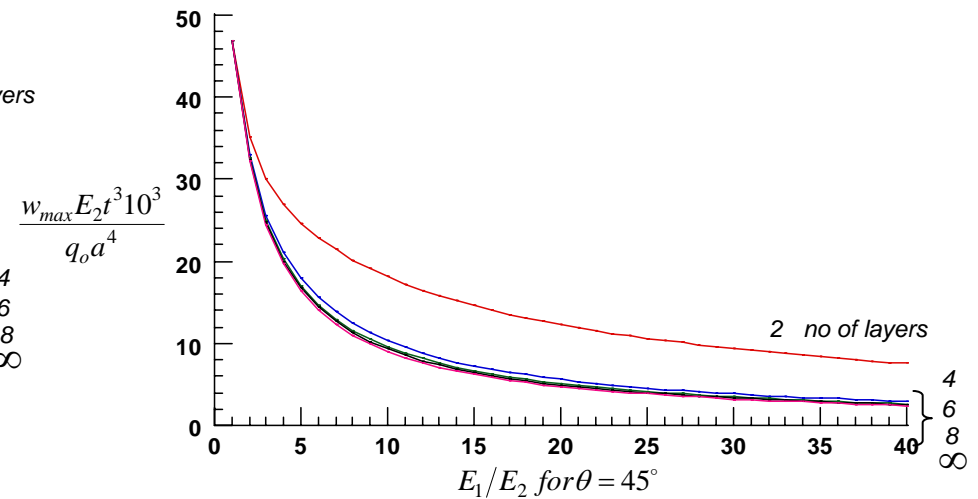
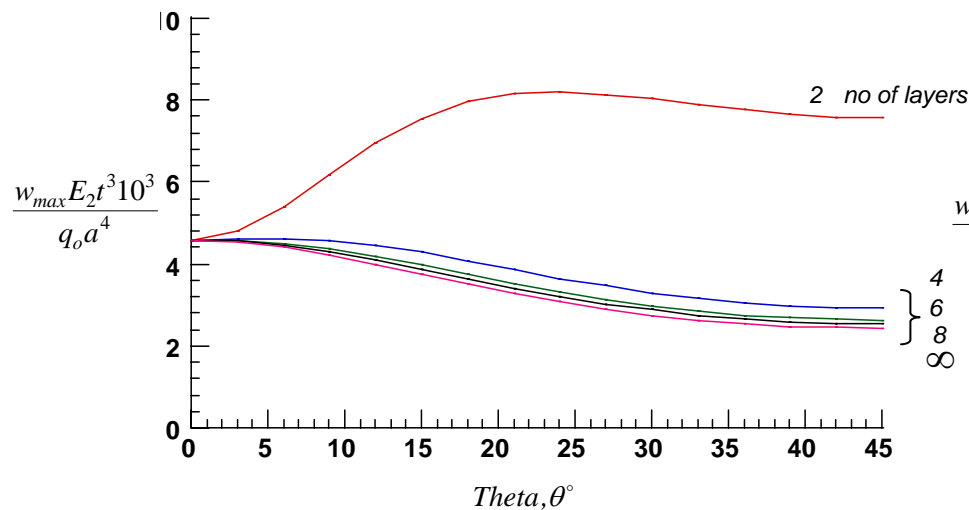
## Solutions – one term solution and for square plate

**Case 2: Uniform loading** 
$$W_{max} = \frac{\alpha q_o a^4}{E_2 t^3} \left( \frac{16}{\pi^2} \right)$$

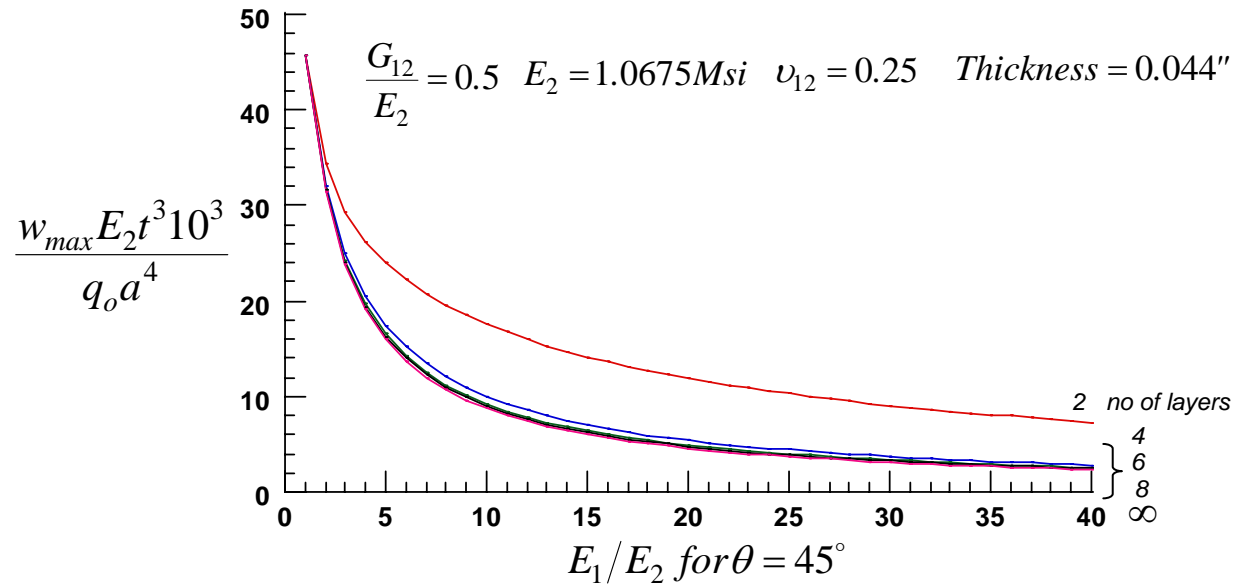
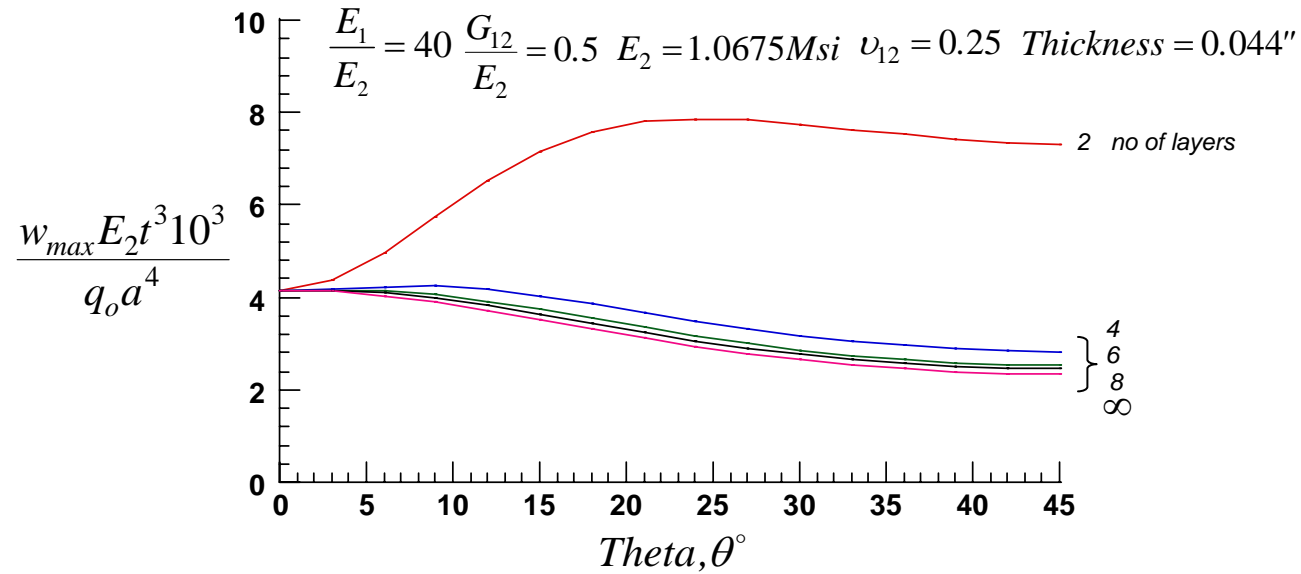
### Example 1: Graphite/epoxy laminate

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### Plots for m=n=1

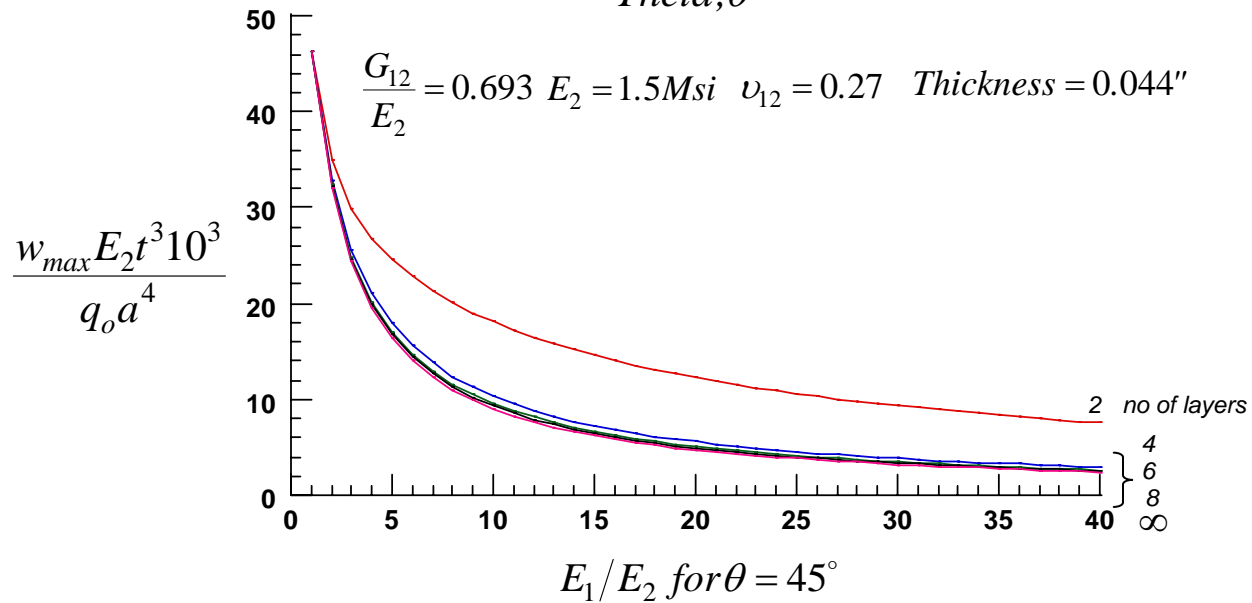
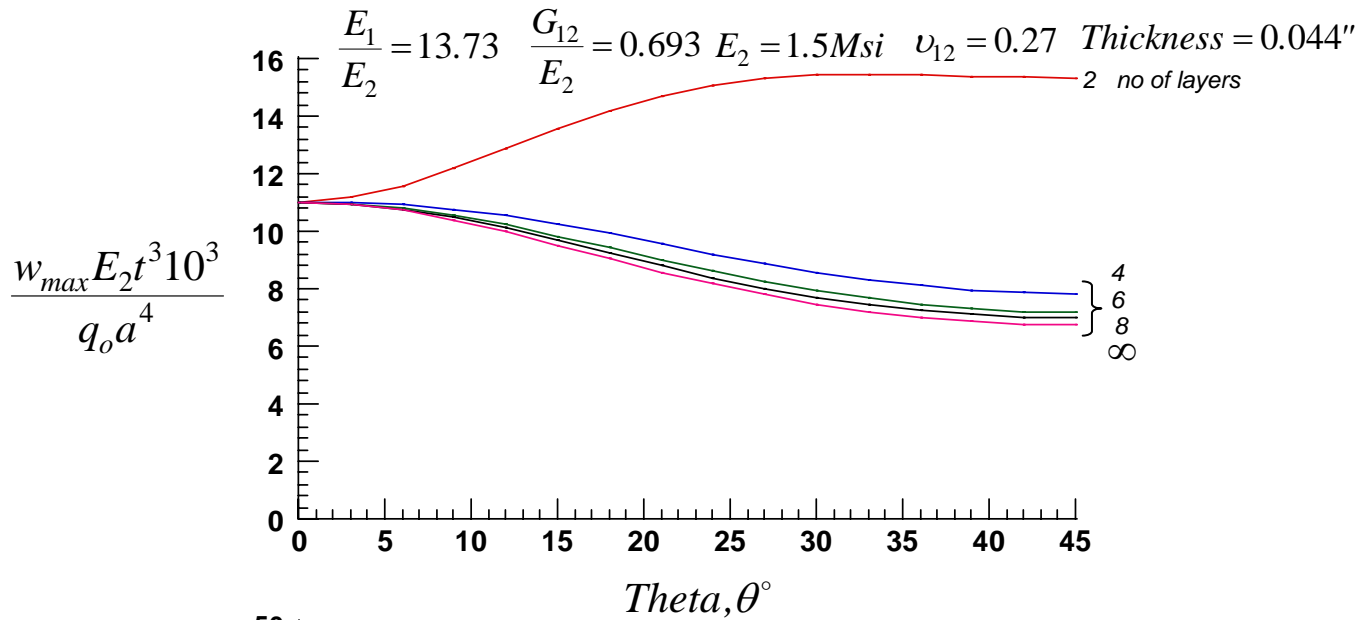


## Multi-term solution (m=n=11) for Case 2-UDL



## Example 2: AS4 Carbon/epoxy laminate with UDL

### Plots for $m=n=1$



## Multi-term solution (m=n=11)

