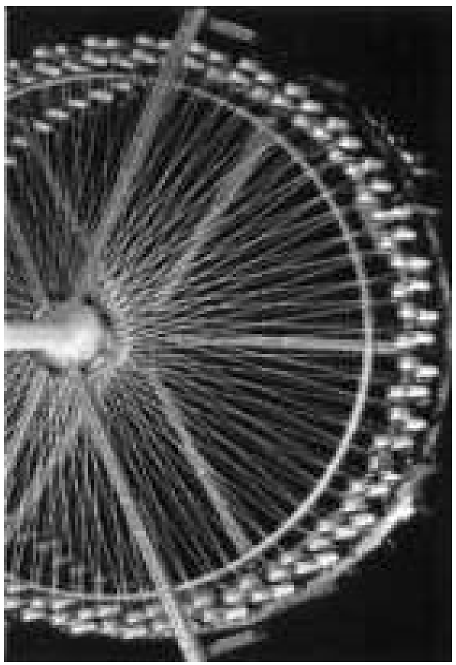
DEVELOP MICROMECHANICS CODE TO PREDICT MECHANICAL PROPERTIES OF TEXTILE FABRIC COMPOSITES

mmTEXlam©



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INTRODUCTION

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MICROMECHANICS OF TEXTILE FABRIC COMPOSITES

- **ANALYTICAL METHODS**

  Chou *et al*
  Ishikawa *et al*
  Ko *et al*
  Pastore *et al*
  Naik *et al*
  Raju, Avva and Foye
  Branch & Shivakumar

- **NUMERICAL METHODS**

  Foye
  Whitcomb
  Glaessegn
  Kelkar

BACKGROUND

- **Simplest and Comprehensive Code**

  TexCAD for 2-D fabric composites

  TEXCAD-3D for 2-D & 3-D fabric composites

- **Limitations of TEXCAD-3D**

  FORTRAN based code

  Procedure-driven code

  No Graphical User Interface

  Valid for impregnated yarns

OBJECTIVES

- **Develop a Graphical User Interfaced, Micromechanics code to analyze unidirectional, 2-D woven and braided, and 3-D braided fabric composites - mmTEXlam**

- **Enhance mmTEXlam code**

- **Verify mmTEXlam by comparing with previous analytical results and test data**

- **Conduct parametric studies to investigate the sensitivity of elastic constants to packing density and yarn count**

ANALYSIS METHODOLOGY

Assumptions:

- **Textile preform architecture are manufactured such that they form a repeating unit cell (RUC) called representative volume element**

- **The unit cell repeats itself in in-plane orthogonal directions**

- **The unit cell consists of yarn elements made up of fiber and matrix and interstitial matrix elements**

- **Fiber volume fraction of the yarn is given by the packing density (p_d) of the unidirectional composite of a similar manufacturing process**

- **Yarn architecture defines the geometric configuration of the yarn in 3-D space and it is unique to unique fabric manufacturing**

Assumptions (contd.)

- **The number of yarns within the unit cell is represented by N**
- **Strain is continuous within the unit cell**
- **Remote uniform strain causes uniform strain both in fiber and matrix. This assumption is called “iso-strain”**
- **Yarn is assumed as transversely isotropic**
- **Matrix is assumed as isotropic**
- **All constituent materials and the composite are assumed to be linear elastic**
- **Deformations are small and linear strain-displacements could be used**

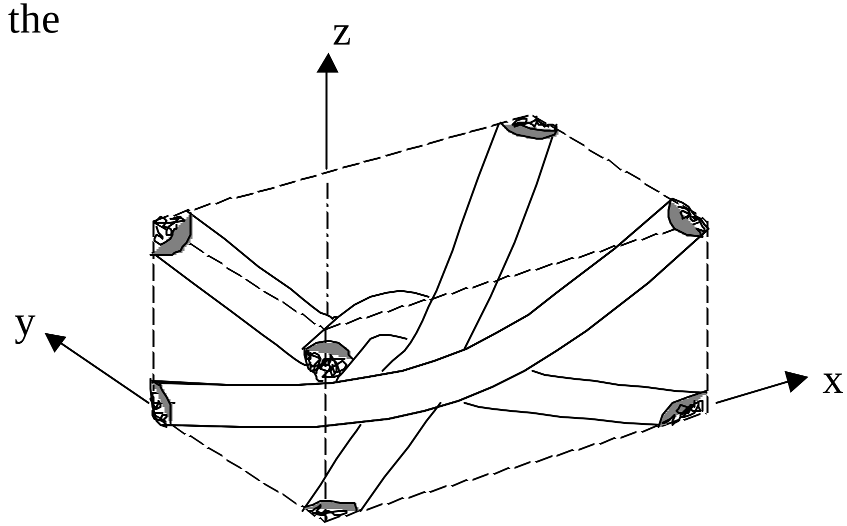
CALCULATION OF THREE-DIMENSIONAL EFFECTIVE STIFFNESS

The Average Stress-Strain Equation of a Unit Cell

The 3-D average stress-strain equation for the composite unit cell is written as

$$\{\bar{s}_{xyz}\} = [\bar{C}_{eff}]\{\bar{e}_{xyz}\}$$

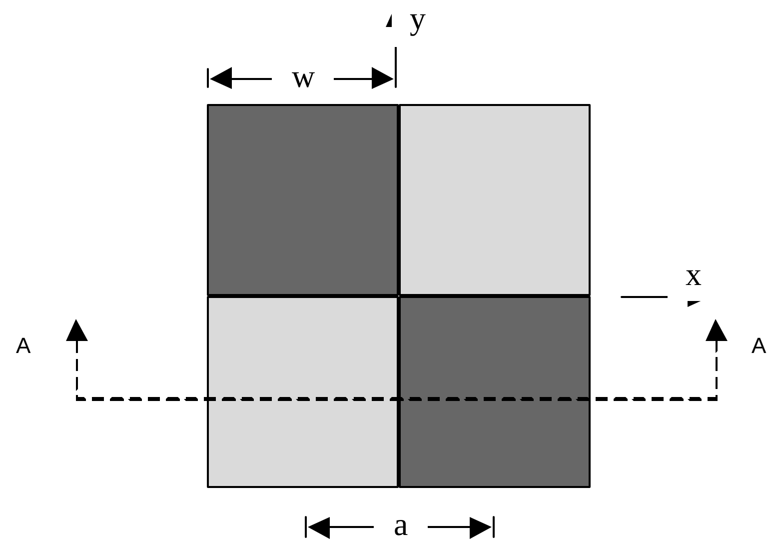
The six stresses and strains are defined in the global coordinate system as follows.



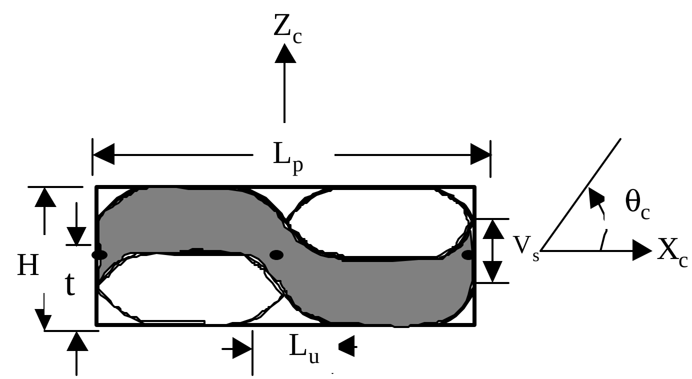
$$\{\bar{s}\} = \begin{Bmatrix} \bar{s}_{xx} \\ \bar{s}_{yy} \\ \bar{s}_{zz} \\ \bar{s}_{yz} \\ \bar{s}_{zx} \\ \bar{s}_{xy} \end{Bmatrix} \qquad \{\bar{e}\} = \begin{Bmatrix} \bar{e}_{xx} \\ \bar{e}_{yy} \\ \bar{e}_{zz} \\ \bar{g}_{yz} \\ \bar{g}_{zx} \\ \bar{g}_{xy} \end{Bmatrix}$$

GEOMETRIC MODELING OF PLAIN WEAVE

A = Yarn cross-sectional area
a = Yarn spacing
d_f = Filament/Fiber diameter
H = RUC thickness
L_p = Projected length of the yarn path
L_u = Undulating length
t = Yarn thickness where t = H/2
V_s = Vertical shift
w = Yarn width
q_c = Crimp angle



$$V_f = \frac{2 p_d A}{Ha} \qquad (1)$$



SECTION A-A

$$A = \frac{p d_f^2 n}{4 p_d} \qquad (2)$$

For a given V_f, the unknown thickness, H, is calculated using Eqs. 1 and 2.

The yarn thickness, t, is related to the RUC thickness, H, by t = H/2. The yarn width, w, is calculated by assuming complete coverage, and is expressed as w = a.

And for the plain weave, V_s = t and the cross-sectional area is also expressed as

$$A = wt - L_u V_s \left(1 - \frac{2}{p}\right) \qquad (3)$$

The unknown parameter, L_u, is calculated using Eqs. 2 and 3.

The undulation in the yarns is often described by it's "crimp angle", which can be expressed as

$$\tan(q_c) = \left| \left(\frac{dZ_c}{dX_c} \right)_{x_c = 0} \right| \qquad (4)$$

where

$$Z_c = \pm \frac{V_s}{2} \sin\left(\frac{p X_c}{L_u} \right) \qquad (5)$$

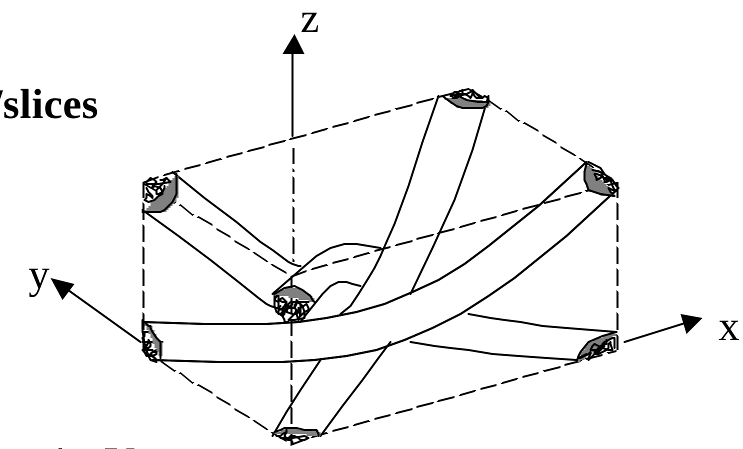
Thus, Eqs. 1 through 5 are used to determine all the required geometry parameters for the plain weave RUC. Similarly, we have geometric definitions corresponding to the various fabric composite architectures.

Constitutive Equations of Yarn and Matrix

- **Unit cell has N number of yarns**
- **Each of the yarn is divided into 'M' parts/slices**
- **Volume fraction of yarns,**

$$V_y = \frac{\text{Fiber Volume fraction}}{\text{Packing Density}} = \frac{V_f}{P_d}$$

- **Volume fraction of interstitial matrix, $V_{im} = 1 - V_y$**
- **Volume fraction of i^{th} yarn's j^{th} slice is V_{yij}**



(a) Constitutive Equation of j^th slice of i^th yarn

The constitutive equation of jth yarn slice in its local coordinate system (x_1, x_2, x_3)

$$\left\{s^{y}_{123}\right\}_{ij} = \left[C^{y}_{123}\right]_{\mathrm{ij}} \left\{e_{123}\right\}$$

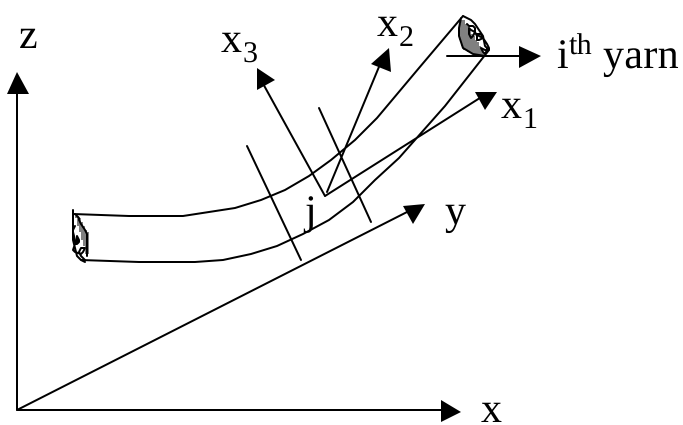
The constitutive matrix $\left[C^{y}_{123}\right]_{\mathrm{ij}}$ is written in terms of the elastic constants of the yarns.

Stresses in the x, y, z coordinate system are derived from the global strains as follows.

$$\left\{s^{y}_{xyz}\right\}_{ij} = \left[T_s\right]\left[C^{y}_{123}\right]_{\mathrm{ij}} \left[T_s\right]^T \left\{e_{xyz}\right\}$$

$$\left\{s^{y}_{xyz}\right\}_{ij} = \left[C^{y}_{xyz}\right]_{\mathrm{ij}} \left\{e_{xyz}\right\}$$

where $\left[C^{y}_{xyz}\right]_{ij} = \left[T_s\right]\left[C^{y}_{123}\right]_{\mathrm{ij}} \left[T_s\right]^T$



(b) Constitutive Equation for Interstitial Matrix

The constitutive equation is independen t of the coordinate system.

$$\{s^{m}_{xyz}\}_{im} = [C^{m}]_{im}\{e_{xyz}\}$$

(c) Constitutive Equation of the Unit Cell

- Iso-strain condition and stress averaging is applied
- Average stress in the unit cell is the weighted sum of stress in all yarn segments and the interstitial matrix
- Weighing factor is directly equal to their volume fraction

Therefore, average stress matrix is

$$\{s_{xyz}\}_{AV} = \sum_{i=1}^{N}\sum_{j=1}^{M} V_{yij}[C^{y}_{xyz}]_{ij}\{e_{xyz}\} + V_{im}[C^{m}]_{im}\{e_{xyz}\}$$

$$\{s_{xyz}\}_{AV} = [\overline{C}_{eff}]\{e_{xyz}\}$$

where $$[\overline{C}_{eff}] = \sum_{i=1}^{N}\sum_{j=1}^{M} V_{yij}[C^{y}_{xyz}]_{ij} + V_{im}[C^{m}]_{im}$$

(d) Elastic Constants of the Composite Lamina

They are computed as follows

Compliance matrix is $\left[\bar{S}\right] = \left[\bar{C}_{eff}\right]^{-1}$

Then the nine elastic constants are as follows

$$E_{xx} = \frac{1}{\bar{S}_{11}} \qquad E_{yy} = \frac{1}{\bar{S}_{22}} \qquad E_{zz} = \frac{1}{\bar{S}_{33}}$$

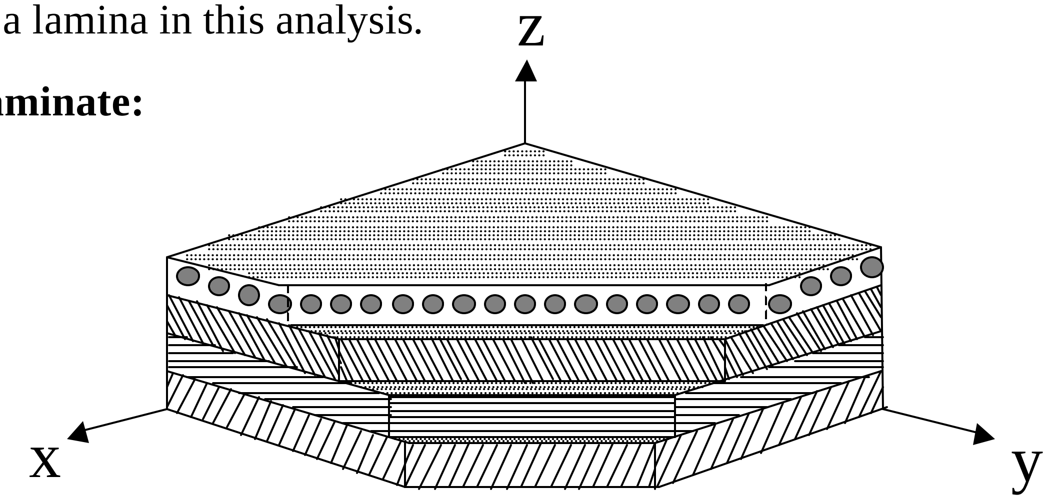
$$n_{xy} = \frac{-\bar{S}_{21}}{\bar{S}_{11}} \qquad n_{yz} = \frac{-\bar{S}_{32}}{\bar{S}_{22}} \qquad n_{zx} = \frac{-\bar{S}_{31}}{\bar{S}_{33}}$$

$$G_{xy} = \frac{1}{\bar{S}_{66}} \qquad G_{yz} = \frac{1}{\bar{S}_{44}} \qquad G_{zx} = \frac{1}{\bar{S}_{55}}$$

TEXTILE COMPOSITE LAMINATES

Procedure

- The thermoelastic properties for each layer can be obtained from the various architecture models as explained earlier.
- For the layered textile composite laminate, it is assumed that classical lamination theory can be used to compute thermal and mechanical properties.
- Each layer of the textile is treated as a lamina in this analysis.



Displacement-Load Equation of a Laminate:

$$\begin{bmatrix} e^{o} \\ \hline k \end{bmatrix} = \left[\begin{array}{c|c} a & b \\ \hline c & d \end{array}\right] \begin{bmatrix} N \\ \hline M \end{bmatrix}$$

where

$$[a] = [A^{-1}] - \{[B^{*}][D^{*-1}]\}[C^{*}]$$

$$[b] = [B^{*}][D^{*-1}]$$

$$[c] = -[D^{*-1}][C^{*}]$$

$$[d] = [D^{*-1}]$$

$$[A^{-1}] = \text{inverse of matrix } [A]$$

$$[B^{*}] = -[A^{-1}][B]$$

$$[C^{*}] = [B][A^{-1}]$$

$$[D^{*}] = [D] - \{[B][A^{-1}]\}[B]$$

Elastic Constants of Textile Laminates

For a symmetric laminate, the bending coupling compliances b_{ij}, with i,j = x, y, s, are zero. Thus, the reference plane strains are related to in-plane forces as follows

$$\begin{bmatrix} e_x^o \\ e_y^o \\ g_s^o \end{bmatrix} = \begin{bmatrix} a_{xx} & a_{xy} & a_{xs} \\ a_{xy} & a_{yy} & a_{ys} \\ a_{sx} & a_{sy} & a_{ss} \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_s \end{bmatrix}$$

where [a] is the extensional laminate compliance matrix.

The laminate may be treated as a homogeneous anisotropic material. The above relation in terms of average laminate stresses is

$$\begin{bmatrix} \bar{s}_x \\ \bar{s}_y \\ \bar{t}_s \end{bmatrix} = \begin{bmatrix} N_x \\ N_y \\ N_s \end{bmatrix} \frac{1}{h}$$

A symmetric laminate may be treated as a homogeneous orthotropic material. Thus, the strain-force relations for the laminate are written in terms of engineering constants and laminate thickness, h, as

$$\begin{bmatrix} e_x^o \\ e_{yx}^o \\ g_s^o \end{bmatrix} = \begin{bmatrix} \frac{1}{\bar{E}_x} & -\frac{\bar{n}_{yx}}{\bar{E}_y} & \frac{\bar{\hbar}_{sx}}{\bar{G}_{xy}} \\ -\frac{\bar{n}_{xy}}{\bar{E}_x} & \frac{1}{\bar{E}_y} & \frac{\bar{\hbar}_{sy}}{\bar{G}_{xy}} \\ \frac{\bar{\hbar}_{xs}}{\bar{E}_x} & \frac{\bar{\hbar}_{sx}}{\bar{E}_y} & \frac{1}{\bar{G}_{xy}} \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_s \end{bmatrix} \frac{1}{h}$$

By equating the corresponding terms in the compliance matrices of [a] and the engineering constants given earlier, we have

$$\bar{\mathrm{E}}_{\mathrm{x}} = \frac{1}{ha_{xx}} \qquad \bar{\mathrm{E}}_{\mathrm{y}} = \frac{1}{ha_{yy}} \qquad \bar{\mathrm{G}}_{\mathrm{xy}} = \frac{1}{ha_{ss}}$$

$$\bar{n}_{\mathrm{xy}} = -\frac{a_{yx}}{a_{xx}} \qquad \bar{n}_{\mathrm{yx}} = -\frac{a_{xy}}{a_{yy}} \qquad \bar{\hbar}_{\mathrm{sx}} = \frac{a_{xs}}{a_{ss}}$$

$$\bar{\hbar}_{\mathrm{xs}} = \frac{a_{sx}}{a_{xx}} \qquad \bar{\hbar}_{\mathrm{ys}} = \frac{a_{sy}}{a_{yy}} \qquad \bar{\hbar}_{\mathrm{sy}} = \frac{a_{ys}}{a_{ss}}$$

ENHANCEMENTS TO ANALYSIS

1. Yarn Properties Computation

- **TexCAD needed yarn properties**
- **Fiber and Matrix properties and p_d of the fabric composite are input**
- **The yarn properties are computed using Modified Hashin's Model**
- **The program has been verified for the analysis on S-Glass, E-Glass and Carbon fiber composites**

ENHANCEMENTS TO ANALYSIS (contd)

2. Micromechanics Models of Unidirectional composites

Introduced the five models

- Mechanics of Materials Model
- Halpin Tsai's Model
- Hashin's Model
- Modified Hashin's Model
- Chamis Model

3. Computation of other properties of Unidirectional Composite

The Chamis model was used to determine properties such as Density, Heat Conductivity, Thermal Capacity, and the Thermal and Moisture expansion coefficients.

DEMONSTRATION OF “mmTEXlam© ”

EVALUATION OF mmTEXlam©

PLAIN WEAVE FABRIC COMPOSITE

- **Composite: Hercules AS4 Graphite Fibers impregnated with 3501-6 Epoxy matrix**

- **The input architecture parameters were**
  p_d = 0.75, d_f = 7 μm, yarn spacing = 1.411 mm, V_f = 64% and yarn size = 3 k

- **Yarn and resin properties used in the analysis are**

Material	E_{11} GPa	E_{22} GPa	G_{12} GPa	ν_{12}	ν_{23}	α_{11} ppm/0 C	α_{22} ppm/0 C
Yarn	144.80	11.73	5.52	0.23	0.30	-0.32	14.00
Resin	3.45	3.45	1.28	0.35	0.35	40.00	40.00

COMPARISON OF mmTEXlam WITH TEXCAD, FEM AND TEST DATA FOR WOVEN FABRIC COMPOSITES

Approach	E_{xx}, E_{yy} GPa	E_{zz} GPa	G_{xz}, G_{yz} GPa	G_{xy} GPa	ν_{xz}, ν_{yz}	ν_{xy}	α_{xx}, α_{yy} ppm/^{0}C	α_{zz} ppm/^{0}C
Plain Weave								
mmTEXlam	64.38	11.49	5.64	4.87	0.396	0.027	1.334	20.71
TexCAD	64.38	11.49	5.64	4.87	0.396	0.027	1.334	20.71
FEM	63.78	11.38	4.97	4.82	0.329	0.031	…	…
Test	61.92	…	…	…	…	0.110	…	…
5-harness Satin Weave								
mmTEXlam	66.33	11.51	4.93	4.89	0.342	0.034	1.462	21.24
TexCAD	66.33	11.51	4.93	4.89	0.342	0.034	1.462	21.24
FEM	65.99	11.38	5.03	4.96	0.320	0.030	…	…
Test	69.43	…	…	5.24	…	0.060	…	…
8-harness Satin Weave								
mmTEXlam	66.81	11.51	4.76	4.89	0.329	0.035	1.494	21.38
TexCAD	66.81	11.51	4.76	4.89	0.329	0.035	1.494	21.38
FEM	66.74	11.45	5.03	4.96	0.320	0.030	…	…
Test	72.19	…	…	6.76	…	0.060	…	…

TWO-DIMENSIONAL TRIAXIAL BRAIDED FABRIC COMPOSITES

The input architecture parameters other than mentioned before were

Braid angle, q = 90º
Fiber count in Axial yarns = 12 k
Fiber count in Braider yarns = 12 k

Study involves three different sets of architecture parameters A1, B1 and B2

2-D Triaxial Braid	Braid Angle, θ	Braider Yarn Size, k	Axial Yarn Size, k	Axial Yarn Spacing, mm	Overall V_f (%)
A1	62.3	12	24	6.10	54.0
B1	67.4	6	18	5.32	51.2
B2	67.9	6	18	5.82	52.0

COMPARISON OF mmTEXlam WITH TEXCAD, FEM AND TEST DATA FOR TWO-DIMENSIONAL TRIAXIAL BRAIDED COMPOSITES

Braid Type	Approach	E_{xx} GPa	E_{yy} GPa	E_{zz} GPa	G_{xy} GPa	G_{xz} GPa	G_{yz} GPa	ν_{xy}	α_{xx} ppm/^{0}C	α_{yy} ppm/^{0}C
A1	mmTEXlam	45.14	40.10	10.42	14.84	4.64	4.13	0.29	1.68	2.65
	TexCAD [2]	45.14	40.10	10.42	14.84	4.64	4.13	0.29	1.68	2.65
	FEM	53.16	46.95	…	…	…	…	0.31	…	…
	Test	45.44	45.64	…	…	…	…	0.31	…	…
B1	mmTEXlam	48.42	42.48	10.13	11.07	4.40	3.90	0.21	1.94	2.90
	TexCAD [2]	48.42	42.48	10.13	11.07	4.40	3.90	0.21	1.94	2.90
	FEM	62.12	47.51	…	…	…	…	0.21	…	…
	Test	44.47	45.16	…	…	…	…	0.19	…	…
B2	mmTEXlam	50.42	42.63	10.23	11.08	4.31	3.90	0.21	1.87	2.89
	TexCAD [2]	50.42	42.63	10.23	11.08	4.31	3.90	0.21	1.87	2.89
	FEM	51.09	43.58	12.07	12.41	…	…	0.21	…	…
	Test	48.47	43.51	…	…	…	…	0.19	…	…

COMPARISON OF mmTEXlam WITH TEXCAD, FEM AND TEST DATA FOR TWO-DIMENSIONAL TRIAXIAL BRAIDED COMPOSITES

For the 3-D 4-Step 5-Row Circular Triaxial braided composite, the input architecture parameters were

Number of Moving Rows = 5

Axial yarn spacing = 0.16 in

Fiber count in Axial Yarn = 12k

Fiber count in Braider yarn = 12k

Planar Braid Angle = 30^0

Fiber Diameter = 0.00028 in

Packing Density = 0.75

Fiber Volume Fraction = 50%

Elastic Constants	mmTEXlam	TEXCAD3D [7]
E_{xx}, GPa	65.96	65.75
E_{yy}, GPa	11.72	11.65
E_{zz}, GPa	9.61	9.57
G_{xy}, GPa	13.20	13.04
G_{xz}, GPa	4.01	4.26
G_{yz}, GPa	3.54	3.64
ν_{xy}	0.85	0.84
ν_{xz}	0.05	0.07
ν_{yz}	0.29	0.30
α_{xx}, ppm/^{0}C	-0.79	-0.84
α_{yy}, ppm/^{0}C	14.50	14.48
α_{zz}, ppm/^{0}C	24.84	24.63

COMPARISON WITH TEST DATA FOR E-GLASS/DERAKANE PLAIN WEAVE COMPOSITE

Material	E_{11}, GPa	E_{22}, GPa	G_{12}, GPa	ν_{12}	ν_{23}
E-Glass fabric	72.50	72.50	30.00	0.20	0.20
Derakane matrix (Vinyl Ester)	3.38		1.41	0.20	

The fabric architecture parameters were
Fiber diameter 10.9 mm **Yarn size 0.824 k**
Yarn spacing 1.59 mm **Fiber volume fraction 42%**

Fabric Material	Yarn size (in k)	Fiber Volume Fraction V_f (in %)	E_{xx}, E_{yy} GPa	ν_{xy}	G_{xy} GPa
E-Glass					
mmTEXlam (p_d = 0.65)	0.824	42	20.72	0.09	4.05
Test	0.824	42	23.03	0.11	3.99

PARAMETRIC STUDY

E-GLASS/DERAKANE VINYL ESTER PLAIN WEAVE COMPOSITE

EFFECT OF PACKING DENSITY ON THE ELASTIC CONSTANTS

Fiber Volume Fraction V_f (in %)	Packing Density p_d	Yarn Size (in k)	mmTEXlam E_{xx}, E_{yy} GPa	E_{zz} GPa	ν_{xy}	ν_{xz}, ν_{yz}	G_{xy} GPa	G_{xz}, G_{yz} GPa
50	0.75	0.824	25.09	12.52	0.097	0.214	5.46	5.35
	0.70	0.824	24.49	11.38	0.090	0.217	4.95	4.87
	0.65	0.824	24.00	10.50	0.084	0.220	4.55	4.51
	0.60	0.824	23.56	9.79	0.079	0.225	4.24	4.24

E-GLASS/DERAKANE VINYL ESTER PLAIN WEAVE COMPOSITE

EFFECT OF YARN COUNT/SIZE ON THE ELASTIC CONSTANTS

Fiber Volume Fraction V_f (in %)	Packing Density p_d	Yarn Size (in k)	mmTEXlam E_{xx}, E_{yy} GPa	E_{zz} GPa	ν_{xy}	ν_{xz}, ν_{yz}	G_{xy} GPa	G_{xz}, G_{yz} GPa
42	0.65	0.824	20.72	9.38	0.087	0.219	4.05	4.01
		1	20.64	9.37	0.087	0.222	4.05	4.04
		3	18.88	9.30	0.077	0.279	4.03	4.71

CONCLUDING REMARKS

- **Developed a Graphical User Interfaced, Micromechanics code to analyze unidirectional, 2-D woven and braided, and 3-D braided fabric composites - mmTEXlam**
- **Enhanced the code mmTEXlam**
- **Verified mmTEXlam code by comparing with previous available analytical and test data**
- **From the parametric studies conducted, it can be concluded that the elastic constants are nearly insensitive to packing density and yarn count**