

# Fuzzy Modal Logics for Societal Theory

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This paper develops fuzzy theories for modeling systems of human or artificial agents. The relevant fuzzy logics are first presented, then these logics are used as a starting point in analyzing social notions. Finally, we show how our fuzzy systems can be used to specify, design, and verify systems of autonomous artificial agents.

Epistemic and deontic logics are modal logics, respectively, of knowledge and of normative concepts. Both have important applications in computer science. We extend the standard definition found in the literature for evaluating formulas in a fuzzy modal logic. The structures that furnish the semantics for modal logics (*Kripke structures*) involve *accessibility relations* between *possible worlds*. In the crisp versions of these logics, various axioms correspond to various properties of these relations. We define these properties for fuzzy accessibility relations and prove various dependencies among these relations. We also prove that some of the correspondences between formulas and properties of these relations carry over to the fuzzy versions of the logics; we also show that some correspondences do not carry over. These correspondences are critical since fuzzy logic does not proceed with axiomatic systems. So we must fall back on the correspondences to distinguish certain formulas as central logical truths. These formulas support dependency networks among assertions in a general way and supply the theoretical support for truth maintenance systems for fuzzy epistemic and deontic reasoning systems.

Our point of departure in societal theory is the philosophical work of Tuomela. This analysis focuses on norms, hence deontic logic is immediately applicable. Two types of norms are recognized, one based on agreement and the other on mutual belief. Agreement rests on common knowledge, which is straightforwardly defined in the standard approaches to epistemic logic and which we treat in our fuzzy versions. So we have a clear application of epistemic logic. Again, belief behaves logically like knowledge except that it does not validate the so-called knowledge axiom, that is, belief

that  $\varphi$  does not entail that  $\varphi$  is true. Indeed, doxastic logic (the logic of belief) is routinely subsumed under epistemic logic by dropping this one axiom. Hence mutual belief can be handled nearly in the same way as common knowledge, and we thus have another application of epistemic logic.

Tuomela assumes crisp notions, but his whole system could be improved by using fuzzy concepts. Knowledge, belief, obligation, permission, joint and individual intentions and actions, commitments, and agreements could be modeled more realistically in a fuzzy manner. We show how our fuzzy versions of epistemic logic more adequately capture the notions of common knowledge and mutual belief that are the basis of norms than do Tuomela's crisp concepts. We also show that norms are properly modeled with concepts from fuzzy deontic logic. Furthermore, group intentions are seen to be fuzzy both with respect to what is intended and with respect to the extent to which various agents are committed to parts of a group task and to the overall task. These aspects of group intentions carry over to group actions.

Finally, we address how our fuzzy modal logics and fuzzy societal theory supply formal methods for the specification, design, and verification of multi-agent systems. An artificial agent is generally seen as something to which we can fruitfully attribute mental concepts. A multi-agent system, in the same vein, is a system of autonomous agents to which we can attribute the social analogues of mental concepts.